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MODELLING THE LINK BETWEEN US INFLATION AND OUTPUT: THE IMPORTANCE OF THE UNCERTAINTY CHANNEL

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Abstract

This article employs an augmented version of the UECCC GARCH specification proposed in Conrad and Karanasos (2010) which allows for lagged in-mean effects, level effects as well as asymmetries in the conditional variances. In this unified framework, we examine the twelve potential intertemporal relationships among inflation, growth and their respective uncertainties using US data. We find that high inflation is detrimental to output growth both directly and indirectly via the nominal uncertainty. Output growth boosts inflation but mainly indirectly through a reduction in real uncertainty. Our findings highlight how macroeconomic performance affects nominal and real uncertainty in many ways and that the bidirectional relation between inflation and growth works to a large extent indirectly via the uncertainty channel.

I Introduction

The nature of the relationship between inflation and output (or unemployment) has been an issue of considerable debate in the macroeconomic literature. While much of the debate has focused on the levels of the two series, there are many theories that highlight the importance of effects due to the interaction between levels and volatilities. For example, Friedman's (1977) famous argument about the negative welfare effects of inflation consists of two claims: higher inflation increases nominal uncertainty, which then decreases output growth.[1] Thus, the negative welfare effects of inflation may (at least partly) work indirectly via nominal uncertainty.

A series of articles published during the last 30 years (see, for example, Logue and Sweeney, 1981; Evans, 1991; Brunner, 1993; Evans and Wachtel, 1993; Ungar and Zilberfarb, 1993; Holland, 1993, 1995; Fuhrer, 1997; Grier and Perry, 1998, 2000; Grier *et al.*, 2004; Elder, 2004; Balcilar and Ozdemir,

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[1] We will use the terms variance, variability, uncertainty and volatility interchangeably in the remainder of the text.

2013) highlights the importance of nominal and real uncertainty for macroeconomic modelling and policy making.

Brunner and Hess (1993) was one of the first articles to employ a univariate GARCH model to test for the first stage of the Friedman hypothesis (see also Baillie *et al.*, 1996). During the last decade, researchers have employed various bivariate GARCH-in-mean models to investigate the relation between the two uncertainties (see, for example, Conrad *et al.*, 2010) and/or to examine their impact on the levels of inflation and growth (see, for example, Elder, 2004; Grier *et al.*, 2004). However, the econometric specifications which are employed in most of these studies are typically characterized by one or more of the following three limitations.

First, the impact from the variabilities on the levels (the so-called in-mean effects) is typically restricted to being contemporaneous (as, for example, in Shields *et al.*, 2005). However, as the theoretical rationale for the in-mean effects usually suggests that it takes some time for them to materialize (e.g. in the Cukierman and Meltzer, 1986, theory it requires a change in monetary policy), it appears more appropriate to investigate such effects within a specification that includes several lags of the variances in the mean equations (see also Elder, 2004 and Conrad *et al.*, 2010).

Second, the existing literature focuses almost exclusively on the impact of macroeconomic uncertainty on performance, but neglects the effects in the opposite direction (level effects). Moreover, the few studies that take level effects into account focus on own but not cross-level effects. In sharp contrast, the empirical results in Logue and Sweeney (1981) suggest that higher nominal uncertainty produces greater variability of real growth. That is, inflation, via the nominal uncertainty channel, affects not only growth (the Friedman hypothesis) but real variability as well. In addition, Brunner (1993) points out that while the second stage of Friedman's hypothesis is plausible, the negative causation between nominal uncertainty and growth could also work in the opposite direction. Therefore, higher growth rates via nominal uncertainty could reduce real variability. In the first stage, high growth rates reduce inflation uncertainty (the Brunner conjecture). In the second stage, this reduced inflation variability lowers real uncertainty (the Logue-Sweeney theory). Thus, a meaningful empirical analysis should allow for bidirectional causality between the four variables.

Third, the two most commonly used specifications are the diagonal constant conditional correlation (CCC) model (see, for example Grier and Perry, 2000; Fountas *et al.*, 2006) and the BEKK representation (see, for example, Shields *et al.*, 2005; Grier and Grier, 2006; Bredin and Fountas, 2009). Both specifications are characterized by rather restrictive assumptions regarding potential volatility interaction. While the CCC model assumes that there is no link between the two uncertainties, the BEKK specification only allows for a positive variance relationship (see Conrad and Karanasos, 2010). In sharp contrast, several economic theories predict either a positive or a negative association between the two volatilities (for more details and a review of the literature, see Arestis *et al.*, 2002; Karanasos and Kim, 2005).

In this study, we investigate the interactions among US inflation, growth and their respective uncertainties using the bivariate unrestricted extended constant conditional correlation (UECCC) GARCH model, defined in Conrad and Karanasos (2010).[2] This model has the advantage that it allows for feedback effects between the two volatilities that can be of either sign, i.e. positive or negative. Further, we augment the UECCC GARCH model in two ways: (1) we estimate a system of equations that allows various lags of the two variabilities to affect the conditional means and (2) we include lagged values of inflation and growth in the two variance specifications and, thereby, control for *own* as well as *cross* level effects. Thus, our model provides a unified empirical framework to test the various economic theories that postulate a relationship between the four variables.

In short, our main results can be summarized as follows. First, inflation is a negative determinant of real growth. This effect takes place both directly and indirectly, via the nominal variability channel, as put forward by Friedman (1977). That is, we find that the impact of inflation on its uncertainty is positive and nominal variability itself has a contemporaneous negative in-mean effect on output growth. Second, we find strong evidence that higher nominal uncertainty increases the average inflation rate. As expected, this effect does not occur contemporaneously but takes 3 months to materialize. If the source for high inflation uncertainty is erratic government policies our findings call for a predictable and rule-based economic policy.

Third, we find that real variability has a positive contemporaneous effect on growth. This finding is in line with the positive correlation between real uncertainty and output growth, which emerges from the model considered in Blackburn and Pelloni (2004) when real shocks predominate. Moreover, higher real uncertainty, with a time delay of 1 month, reduces inflation. Both results show that the behavior of macroeconomic performance is influenced by its volatility, but also that the significance and the sign of the in-mean effects depend on the correct modelling of the lag length.

Further, of significant relevance is our finding that inflation has a positive impact on real uncertainty, as predicted by Dotsey and Sarte (2000). We also find that growth affects inflation variability negatively, thus supporting the Brunner (1993) conjecture. The potential for reverse causation to have influenced the nominal uncertainty-growth link has not yet been considered in the literature. Our results suggest the importance of paying explicit attention to the effects of macroeconomic performance on its variability.

Dotsey and Sarte (2000) highlight the fact that the volatilities of inflation and growth are directly linked and that this observation deserves empirical attention since it has important implications for the analysis of the impact of macroeconomic performance on real uncertainty. Regarding this relation, our results are strongly in favour of the prediction by Logue and Sweeney (1981)

[2] The specification is termed 'unrestricted extended' because it can be viewed as an unrestricted version of the extended CCC (ECCC) specification of Jeantheau (1998), which allows for positive volatility feedback only.

that higher nominal variability increases real uncertainty. Therefore inflation through its variability affects (1) growth negatively as predicted by Friedman (1977) and (2) real uncertainty positively as predicted by Dotsey and Sarte (2000). In other words not only does the Friedman hypothesis consist of two stages but so does the Dotsey and Sarte conjecture as well. The first stage is identical in both whereas the second stage of the latter is the Logue and Sweeney theory.

The remainder of the article is organized as follows. Section II introduces the bivariate UECCC GARCH model, presents its properties and sets out assumptions and notation. In section III we present a brief overview of the theories that link inflation, growth and their respective uncertainties. In section IV we present and discuss the significance of the empirical results. Section V compares our findings to the results in the previous literature. In section VI we present a sensitivity analysis of our results with respect to the specification of the model, subsamples and the data frequency. Finally, section VII concludes the article.

II The Bivariate GARCH Model

We use a bivariate model to simultaneously estimate the conditional means, variances and covariances of inflation and output growth. Let $\mathbf{y}_t = [\pi_t y_t]'$ represent the 2×1 vector with the inflation rate and real output growth. Further, $\mathcal{F}_{t-1} = \sigma(\mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \ldots)$ is the filtration generated by the information available up through time $t-1$ and $\mathbf{h}_t = [h_{\pi,t} h_{y,t}]'$ denotes the vector of $\mathcal{F}_{t-1}$ measurable conditional variances. We estimate the following bivariate AR(p)-GARCH(1,1)-in-mean model

$$\mathbf{y}_t = \mathbf{\Gamma}_0 + \sum_{l=1}^{p} \mathbf{\Gamma}_l \mathbf{y}_{t-l} + \sum_{r=0}^{s} \mathbf{\Delta}_r \mathbf{h}_{t-r} + \boldsymbol{\varepsilon}_t, \tag{1}$$

where $\mathbf{\Gamma}_0 = [\gamma_i]_{i=\pi,y}$, $\mathbf{\Gamma}_l = [\gamma_{ij}^{(l)}]_{i,j=\pi,y}$ and $\mathbf{\Delta}_r = [\delta_{ij}^{(r)}]_{i,j=\pi,y}$. Let $\mathbf{I}$ be the 2×2 identity matrix and L the lag operator. We assume that the roots of $\left|\mathbf{I} - \sum_{l=1}^{p} \mathbf{\Gamma}_l L^l\right|$ lie outside the unit circle. Note that our specification allows the conditional variances to affect the level variables contemporaneously and up to lag $s > 0$. Controlling for both autoregressive terms as well as lagged conditional variances is important, because, as shown in Ghysels *et al.* (2005) and Conrad and Karanasos (2014), the omission of autoregressive terms/lagged conditional variances may lead to spuriously significant in-mean/autoregressive terms.[3]

The residual vector is defined as $\boldsymbol{\varepsilon}_t = [\varepsilon_{\pi,t} \varepsilon_{y,t}]' = \mathbf{z}_t \odot \mathbf{h}_t^{\wedge 1/2}$, where the symbols $\odot$ and $^\wedge$ denote the Hadamard product and the element-wise exponentiation respectively. The stochastic vector $\mathbf{z}_t = [z_{\pi,t} z_{y,t}]'$ is assumed to be independently and identically distributed (*i.i.d.*) with mean zero, finite second moments and 2×2 correlation matrix $\mathbf{R} = [\rho_{ij}]_{i,j=\pi,y}$ with diagonal elements

[3] In section VI we also consider a specification in which the mean is a function of the conditional standard deviations, i.e. $\mathbf{h}_{t-r}^{\wedge 1/2}$, instead of the conditional variances.

equal to one and off-diagonal elements absolutely less than one. Thus, we have $\mathbf{E}[\boldsymbol{\varepsilon}_t|\mathcal{F}_{t-1}] = \mathbf{0}$ and $\mathbf{H}_t = \mathbf{E}[\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'|\mathcal{F}_{t-1}] = diag\{\mathbf{h}_t\}^{1/2}\mathbf{R}diag\{\mathbf{h}_t\}^{1/2}$, where $h_{\pi y,t} = \rho_{\pi y}\sqrt{h_{\pi,t}h_{y,t}}$ is the conditional covariance.

Following Conrad and Karanasos (2010), we impose the UECCC GARCH (1,1) structure on the conditional variances:

$$\mathbf{h}_t = \boldsymbol{\omega} + \mathbf{A}\boldsymbol{\varepsilon}_{t-1}^{\wedge 2} + \mathbf{B}\mathbf{h}_{t-1}, \tag{2}$$

where $\boldsymbol{\omega} = [\omega_i]_{i=\pi,y}$, $\mathbf{A} = [a_{ij}]_{i,j=\pi,y}$ and $\mathbf{B} = [b_{ij}]_{i,j=\pi,y}$. We assume that the above model is minimal in the sense of Jeantheau (1998, Definition 3.3) and invertible (see Assumption 2 in Conrad and Karanasos, 2010). The invertibility condition implies that the inverse roots of $|\mathbf{I}-\mathbf{B}L|$, denoted by ϕ_1 and ϕ_2, lie inside the unit circle. Following Conrad and Karanasos (2010) we also impose the four conditions which are necessary and sufficient for $\mathbf{h}_t > 0$ for all t: (i) $(1-b_{yy})\omega_\pi + b_{\pi y}\omega_y > 0$ and $(1 - b_{\pi\pi})\omega_y + b_{y\pi}\omega_\pi > 0$, (ii) ϕ_1 is real and $\phi_1 > |\phi_2|$, (iii) $\mathbf{A} \geq 0$ and (iv) $[\mathbf{B} - \max(\phi_2,0)\mathbf{I}]\mathbf{A} > 0$. Note that these constraints do not place any *a priori* restrictions on the signs of the coefficients in the **B** matrix. In particular, this implies that negative volatility spillovers are possible.

The UECCC specification nests the diagonal CCC model when **A** and **B** are diagonal matrices and Jeantheau's (1998) ECCC model when $a_{ij} \geq 0$ and $b_{ij} \geq 0$. Arestis *et al*. (2002) and Arestis and Mouratidis (2004) correctly argue that any multivariate GARCH model which imposes positive volatility feedback cannot be used to estimate and test for a volatility trade-off.[4] However, although this is true for both the BEKK representation and the restricted ECCC specification of Jeantheau (1998), this is no longer the case for the unrestricted version of the latter formulation. More specifically, the necessary and sufficient conditions derived in Conrad and Karanasos (2010) ensure the positive definiteness of the conditional covariance matrix even in the case of negative volatility feedback. While negative values of the GARCH coefficients have commonly been thought of as resulting either from sampling error or model misspecification, they show that this is not necessarily the case. Interestingly, negative volatility spillovers may be in line with economic theory (see section III).

Finally, we augment the variance specification to allow for asymmetries and to include level effects:

$$\mathbf{h}_t = \boldsymbol{\omega} + (\mathbf{A} + \mathbf{G}\cdot\mathbf{1}_{\{\boldsymbol{\varepsilon}_{t-1} > 0\}})\boldsymbol{\varepsilon}_{t-1}^{\wedge 2} + \mathbf{B}\mathbf{h}_{t-1} + \mathbf{e}^{\wedge\boldsymbol{\Lambda}\mathbf{y}_{t-1}}, \tag{3}$$

where $\mathbf{G}=[g_{ij}]_{i,j=\pi,y}$, $\mathbf{1}_{\{\boldsymbol{\varepsilon}_t > 0\}} = [1_{\{\varepsilon_{i,t} > 0\}}]_{i=\pi,y}$ is an indicator function and $\boldsymbol{\Lambda} = [\lambda_{ij}]_{i,j=\pi,y}$. We choose the exponential specification for the level effects for two reasons. First, it ensures that our non-negativity conditions are still sufficient for guaranteeing positive conditional variances. Second, economic theory suggests that the positive impact of inflation on uncertainty should

[4] As an alternative, it is possible to use either a stochastic volatility or an EGARCH model, both of which assume an exponential specification of the conditional variance and, thereby, allow us to estimate the model parameters without any positivity restrictions.

increase as the level of inflation rises (see Ungar and Zilberfarb, 1993).[5] Note that we can easily control for further lags of the level effect by adding the respective terms to equation (3). An alternative approach to introducing level effects in an exponential fashion as in equation (3) would be by adding the lagged inflation rates either linearly (see Conrad *et al.*, 2010) or quadratically (see Brunner and Hess, 1993). In the following, we will term the asymmetric GARCH (AGARCH) in-mean-level (ML) model AGARCH-ML.

Finally, it is important to highlight the differences between the model employed in this study and the specification used in Conrad *et al.* (2010) to analyze UK data. First, the model introduced above allows for in-mean effects of nominal and real uncertainty at different lags. Second, we specify the level effects in an exponential fashion. As explained above, this specification does not only ensure the positivity of the conditional variances but is also in line with economic theory. Third, we explicitly allow for asymmetries in the conditional variances. In section IV we will show that all three issues are highly relevant empirically.

III Economic Theories

Since intensive discussions of the economic theories that rationalize a relationship between inflation, output growth and their respective uncertainties can be found in, e.g. Karanasos and Kim (2005), Fountas *et al.* (2006), Fountas and Karanasos (2007, 2008) and Conrad *et al.* (2010), in Table 1 we only provide a brief summary of those theories that will be tested (and confirmed) in section IV.

Table 1 shows that the different variables can have *direct* as well as *indirect* effects. For example, the first stage of the Friedman (1977) hypothesis states that higher inflation leads to an increase in nominal uncertainty ($\pi \xrightarrow{+} h_\pi$) and, hence, can be considered as a direct effect. On the other hand, the Dotsey and Sarte (2000) conjecture, which postulates that more inflation increases real uncertainty, relies on the combination of the first stage of the Friedman (1977) hypothesis with the positive impact of nominal uncertainty on real variability ($h_\pi \xrightarrow{+} h_y$) as implied by the Logue and Sweeney (1981) theory. That is, the Dotsey and Sarte (2000) conjecture should be considered as an indirect effect which works via nominal uncertainty ($\pi \xrightarrow{+} h_\pi \xrightarrow{+} h_y$).

In addition, Table 1 makes clear that most theories can only be tested within our flexible UECCC-GARCH framework. For example, a test of the Fuhrer (1997) theory requires a model which allows for negative volatility spillovers. Similarly, the Brunner (1993) conjecture ($y \xrightarrow{-} h_\pi$) can only be tested within a model with cross-level effects.

[5] Notice that in equation (3) the derivative of $h_{\pi,t}$ with respect to π_{t-1} is given by $\lambda_{\pi\pi} e^{\lambda_{\pi\pi}\pi_{t-1}}$, which, if $\lambda_{\pi\pi} > 0$, increases as π_{t-1} rises.

Table 1
Summary of economic theories

	π	y	h_π	h_y
π		Phillips curve: $y \overset{+}{\rightarrow} \pi$	$\frac{\text{Cukierman and Meltzer (1986): } h_\pi \overset{+}{\rightarrow} \pi}{\text{Holland (1995): } h_\pi \overset{-}{\rightarrow} \pi \text{(conjecture)}}$	Fuhrer (1997)/Cukierman and Meltzer (1986): $h_y \overset{-}{\rightarrow} h_\pi \overset{+}{\rightarrow} \pi$
y	Gillman and Kejak (2005): $\pi \overset{-}{\rightarrow} y$		Friedman (1977), 2nd stage: $h_\pi \overset{-}{\rightarrow} y$	Blackburn (1999); Blackburn and Pelloni (2005): h_y
h_π	Friedman (1977), 1st stage: $\pi \overset{+}{\rightarrow} h_\pi$	Brunner(1993): $y \overset{-}{\rightarrow} h_\pi$(conjecture)		Fuhrer (1997): $h_y \overset{-}{\rightarrow} h_\pi$
h_y	Dotsey and Sarte (2000): $\pi \overset{+}{\rightarrow} h_\pi \overset{+}{\rightarrow} h_y$ (conjecture)	Brunner (1993)/Logue and Sweeney (1981): $y \overset{-}{\rightarrow} h_\pi \overset{+}{\rightarrow} h_y$	$\frac{\text{Logue and Sweeney (1981)}: h_\pi \overset{+}{\rightarrow} h_y}{\text{b29Fuhrer (1997)}: h_\pi \overset{-}{\rightarrow} h_y}$	

Notes: The effects of inflation (nominal uncertainty) on the other three variables are presented in the first (third) column. The effects of growth (real variability) on the other three variables are presented in the second (fourth) column. +/−: the effect is positive/negative.

IV Empirical Results

We employ deseasonalized monthly data obtained from the FRED database at the Federal Reserve Bank of St. Louis. The annualized inflation and output growth series are calculated as 1200 times the monthly difference in the natural log of the Consumer Price Index and the Industrial Production Index respectively. The data range from 1960:01 to 2010:01 and, hence, comprise 600 usable observations. Applying various unit root tests to both series, we came to the conclusion that inflation as well as output growth can be treated as stationary variables.

The parameter estimates of the UECCC GARCH-in-mean model are obtained by quasi-maximum likelihood estimation. The lag orders in equation (1) were chosen on the basis of likelihood ratio (LR) tests and information criteria. In the inflation/output equations the best model includes 12/4 lags of inflation/output. Significant cross effects between inflation and output are found at lags 2 and 3 only. For reasons of brevity, we refrain from presenting the estimation results for the autoregressive parameters. Instead, in Table 2 we concentrate on the main parameters of interest. Finally, we also tested the constant conditional correlation assumption using Tse's (2000) test (not reported). However, we did not find evidence for dynamic conditional correlations.[6]

Baseline specification

First, from equations (4) and (5) in Table 2 there is strong evidence for bidirectional feedback between the levels of inflation and output growth. More specifically, with a delay of 2 months inflation affects growth negatively, whereas growth has a positive effect on inflation after 3 months. The first observation is in line with the implications of the different theories summarized in Gillman and Kejak (2005) and the empirical findings presented in, e.g. Temple (2000) and Barro (2001). The latter observation supports the standard Phillips curve, which suggests that, at least over the short run, high output growth (low unemployment) leads to increasing inflation (for empirical evidence see, e.g. Stock and Watson, 1999).

Second, the two variance expressions in equation (6) in Table 2 allow us to analyze the potential spillover effects between the two volatilities. The coefficients $a_{\pi y}$, $b_{\pi y}$ and b_{yy} were found to be insignificant and, hence, excluded from the specification. That is, inflation uncertainty obeys a GARCH(1,1) structure, while real variability is best characterized as an ARCH(1) process. Because $a_{y\pi}$ and $b_{y\pi}$ are significantly estimated, both squared inflation residuals and nominal volatility affect real uncertainty but not vice versa.[7] Since $b_{y\pi}$

[6] Initially we also experimented with DCC type models, but again found no evidence for dynamic correlations.

[7] More precisely, squared inflation residuals $\varepsilon^2_{\pi,t-1}$ have a direct effect (through $a_{y\pi}$) on output uncertainty $h_{y,t}$. They also have an indirect effect by increasing $h_{\pi,t}$ (through $a_{\pi\pi}$) and thereby $h_{y,t+1}$ (through $b_{y\pi}$) in the next period (and thereafter because $h_{\pi,t}$ is persistent). Also note that the conditional heteroskedasticity in growth is mainly due to the transmission of the conditional heteroskedasticity from inflation.

Table 2
Bivariate AR-UECCC-GARCH(1,1)-in-mean model

$$\pi_t = \ldots + \underset{(0.011)}{0.025}\, y_{t-3} \ldots + \underset{(0.016)}{0.032}\, h_{\pi,t-3} - \underset{(0.0008)}{0.0018}\, h_{y,t-1} + \varepsilon_{\pi,t} \quad (4)$$

$$y_t = \ldots - \underset{(0.087)}{0.359}\, \pi_{t-2} \ldots - \underset{(0.062)}{0.226}\, h_{\pi,t} + \underset{(0.010)}{0.030}\, h_{y,t} + \varepsilon_{y,t} \quad (5)$$

$$\begin{pmatrix} h_{\pi,t} \\ h_{y,t} \end{pmatrix} = \begin{pmatrix} \underset{(0.249)}{0.985} \\ \underset{(3.895)}{35.785} \end{pmatrix} + \begin{pmatrix} \underset{(0.044)}{0.303} & - \\ \underset{(0.282)}{0.839} & \underset{(0.059)}{0.301} \end{pmatrix} \begin{pmatrix} \varepsilon^2_{\pi,t-1} \\ \varepsilon^2_{y,t-1} \end{pmatrix} + \begin{pmatrix} \underset{(0.055)}{0.592} & - \\ \underset{(0.478)}{0.727} & - \end{pmatrix} \begin{pmatrix} h_{\pi,t-1} \\ h_{y,t-1} \end{pmatrix} \quad (6)$$

$$h_{\pi y,t} = \underset{(0.044)}{0.044} \sqrt{h_{\pi,t} h_{y,t}} \quad (7)$$

Residual diagnostics

	$Q(4)$	$Q^2(4)$	$Q(10)$	$Q^2(10)$
Inflation eq.	3.21 [0.52]	1.95 [0.75]	9.41 [0.49]	7.44 [0.68]
Output eq.	4.90 [0.30]	1.86 [0.76]	9.14 [0.52]	12.13 [0.28]

Notes: The table reports the quasi-maximum likelihood parameter estimates of the bivariate AR (p)-UECCC-GARCH(1,1)-in-mean model for the US inflation (π_t) and output growth (y_t) data. $h_{\pi,t}$ and $h_{y,t}$ denote the conditional variances of inflation and output respectively. The numbers in parentheses are robust standard errors, the numbers in brackets are p-values. $Q(s)$ and $Q^2(s)$ are the Ljung-Box tests for sth-order serial correlation in the standardized and squared standardized residuals.

is positive and significant there is strong evidence that nominal uncertainty has a positive impact on real volatility, as predicted by Logue and Sweeney (1981). In contrast to the implication of the Fuhrer (1997) theory, we do not find evidence for a significant *direct* impact in the opposite direction. Note that the parameter restrictions established in Conrad and Karanasos (2010) are naturally satisfied, since all ARCH/GARCH parameters are positive.

Next, we discuss the parameter estimates of the in-mean terms in equations (4) and (5) in Table 2. Whether higher nominal uncertainty increases or decreases inflation depends on the central bank's reaction function. If a central bank is sufficiently independent and primarily focused on achieving price stability, the central bank will react to higher nominal variability by reducing the inflation rate. In the words of Holland (1995, p. 832): 'one possible reason for greater nominal variability to precede lower inflation is that an increase in uncertainty is viewed by policymakers as costly, inducing them to reduce inflation in the future'. If, on the other hand, the central bank is targeting inflation as well as output growth, then the reaction of the central bank will depend on the respective weights that are given to the two targets. If the weight on growth is sufficiently large, the central bank has an incentive to increase inflation in the presence of higher nominal uncertainty (see Cukierman and Meltzer, 1986). The in-mean parameter estimate in equation (4), $\delta^{(3)}_{\pi\pi}$, suggests that

– with a lag of 3 months – higher nominal uncertainty indeed leads to more inflation in the United States. This finding is in line with the observation that the Fed is targeting both inflation and growth and, hence, suggests that across our sample considerable weight has been given to the latter.[8]

The finding that $\delta_{y\pi}^{(0)}$ is negative and significant in equation (5) supports the second stage of the Friedman (1977) hypothesis that increasing inflation uncertainty leads to lower investment (see also Pindyck, 1991) and affects output growth negatively. Interestingly, the two in-mean effects of real uncertainty are also significant. With a delay of 1 month, increasing output volatility appears to lower the average inflation rate ($\delta_{\pi y}^{(1)}$ in equation (4) is negative). In addition, higher real variability appears to increase output growth ($\delta_{yy}^{(0)}$ is positive and significant in equation (5) in Table 2). This finding is consistent with the theoretical predictions in Blackburn and Pelloni (2005), who study the relation between output growth and its variability in a stochastic monetary growth model.

It is important to highlight again that the effects from the two uncertainties on inflation arise with some time delay (insignificant contemporaneous parameters are not presented), which is to be expected given the economic theories and the fact that we work with monthly data.[9]

We also investigated the effect of omitting the autoregressive terms from the mean equations. In this case the impact of nominal uncertainty on inflation is estimated to be considerably stronger. On the other hand, the effect of real uncertainty on growth becomes insignificant. These changes can be explained by the positive/negative relation between lagged inflation/growth and nominal/real uncertainty (discussed in more detail below). If the autoregressive terms are omitted, this generates a sort of 'omitted variables bias'. Similarly, omitting the lagged conditional variances from the mean equations leads to biased estimates of the autoregressive terms. Thus, it is important to control for both (see Conrad and Karanasos, 2014).

Finally, note that from the Ljung-Box tests it appears that our model is well specified, i.e. there is no evidence for serial correlation in the standardized and squared standardized residuals at lags 4 and 10. The finding that the constant conditional correlation $\rho_{\pi y}$ is not significantly different from zero is in line with Grier and Perry (2000). However, it is important to note that in our bivariate UECCC-GARCH(1,1) model, given by equation (2), the two conditional variances will be correlated even if $\rho_{\pi y}$=0 due to the presence of volatility and residuals spillovers. That is, even if $\rho_{\pi y}$=0 the UECCC GARCH

[8] There is a controversy regarding whether the sign of the response of inflation to an increase in nominal uncertainty can be related to measures of central bank independence. While the evidence in Grier and Perry (1998) suggests such a link, the results in Conrad and Karanasos (2005) and Fountas and Karanasos (2007) do not support this hypothesis. Similarly, Ciccarelli and Mojon (2010) provide evidence that country specific inflation is largely driven by a global inflation factor. Among other things, this global factor reflects which monetary strategies are globally dominant at a certain point in time.

[9] In the previous studies which employed GARCH-in-mean models the uncertainties were restricted to affecting the levels contemporaneously, often resulting in insignificant parameter estimates (see section V).

model with spillovers does not reduce to two separate GARCH equations that could be estimated individually.

The model with asymmetries and level effects

Next, we reestimate the model and allow for asymmetries as well as level effects, i.e. we estimate our model with the augmented variance specification given by equation (3). The results are presented in Table 3.[10]

We first discuss the estimates for the level coefficients λ_{ij}, $i,j = \pi,y$. The coefficient estimate, $\lambda_{\pi\pi} > 0$, indicates that higher lagged inflation tends to increase nominal uncertainty, thus supporting the first stage of the Friedman (1977) theory. Since $\lambda_{y\pi} > 0$, there is also strong evidence that higher inflation increases the variability in output growth.[11] Next, we turn to the effects of growth on the two volatilities. We find that negative growth rates increase nominal uncertainty.[12] The coefficient estimate, $\lambda_{\pi y} < 0$, is in line with the

Table 3
Bivariate model with asymmetry and level effects

$$\pi_t = \ldots + \underset{(0.011)}{0.011}\, y_{t-3} \ldots + \underset{(0.019)}{0.033}\, h_{\pi,t-3} - \underset{(0.0019)}{0.0031}\, h_{y,t-1} + \varepsilon_{\pi,t} \tag{8}$$

$$y_t = \ldots - \underset{(0.100)}{0.249}\, \pi_{t-2} \ldots - \underset{(0.090)}{0.270}\, h_{\pi,t} + \underset{(0.009)}{0.033}\, h_{y,t} + \varepsilon_{y,t} \tag{9}$$

$$\begin{aligned}\begin{pmatrix} h_{\pi,t} \\ h_{y,t} \end{pmatrix} &= \begin{pmatrix} \underset{(0.253)}{-0.265} \\ \underset{(3.644)}{28.939} \end{pmatrix} + \begin{pmatrix} \underset{(0.039)}{0.259} & - \\ \underset{(0.282)}{0.626} & \underset{(0.119)}{0.467} \end{pmatrix} \begin{pmatrix} \varepsilon^2_{\pi,t-1} \\ \varepsilon^2_{y,t-1} \end{pmatrix} \\ &+ \begin{pmatrix} - & - \\ - & -\underset{(0.123)}{0.322} \end{pmatrix} \begin{pmatrix} \mathbf{1}_{\{\varepsilon_{\pi,t-1} > 0\}} \varepsilon^2_{\pi,t-1} \\ \mathbf{1}_{\{\varepsilon_{y,t-1} > 0\}} \varepsilon^2_{y,t-1} \end{pmatrix} + \begin{pmatrix} \underset{(0.072)}{0.508} & - \\ \underset{(0.574)}{1.372} & \end{pmatrix} \begin{pmatrix} h_{\pi,t-1} \\ h_{y,t-1} \end{pmatrix} \\ &+ \begin{pmatrix} \exp(\underset{(0.032)}{0.105}\, \pi_{t-1}) & -\underset{(0.084)}{0.201}\, \mathbf{1}_{\{y_{t-1} < 0\}} y_{t-1} \\ \exp(\underset{(0.081)}{0.205}\, \pi_{t-1}) & - \end{pmatrix}\end{aligned} \tag{10}$$

$$h_{\pi y,t} = \underset{(0.043)}{0.031} \sqrt{h_{\pi,t} h_{y,t}} \tag{11}$$

Residual diagnostics

	$Q(4)$	$Q^2(4)$	$Q(10)$	$Q^2(10)$
Inflation eq.	6.66 [0.16]	2.83 [0.59]	15.65 [0.11]	10.84 [0.37]
Output eq.	5.12 [0.27]	1.36 [0.85]	8.79 [0.55]	16.02 [0.10]

Notes: See Table 2.

[10] Note that ω_π is estimated to be negative but insignificant. In the UECCC GARCH model (without level effects) the non-negativity constraints can be satisfied if one or even both constants are negative. Since it is insignificant, we treat ω_π as being zero when checking the constraints.

[11] Both findings are in line with the results that we obtained in Conrad *et al.* (2010) using UK data.

[12] Since we found no significant effect for positive growth rates, we employed a specification with $\mathbf{1}_{\{y_{t-1} > 0\}} y_{t-1}$.

prediction by Brunner (1993, p.514) 'that when economic activity falls off, there is some uncertainty generated about the future path of monetary policy, and consequently, about the future path of inflation'.

As can be seen from Table 3, while there is no evidence for asymmetries in nominal uncertainty, there is strong asymmetry in real variability. The parameter estimate in equation (13), $g_{yy} < 0$, shows that real variability is to a large extent driven by negative residuals, i.e. $\varepsilon_{y,t-1} < 0$, in the growth equation.

Note that our conclusions regarding the volatility feedback and the impact of macroeconomic uncertainty on performance are qualitatively not affected by including level effects and asymmetries. That is, (1) nominal uncertainty has a positive impact on real variability and (2) the own in-mean effects are positive whereas the cross in-mean effects are negative. On the other hand, while inflation still affects growth negatively, the *direct* impact of growth on inflation disappears when we appropriately control for level effects (that is $\gamma_{\pi y}^{(3)}$ becomes insignificant in equation (8) in Table 3).

Within the full model it is now possible to test for the existence of *indirect* effects which work via the level effects or the uncertainty channel or a combination of both. We test for these indirect effects by means of LR tests.

First, we use a LR statistic to test for the joint significance of both stages of the Friedman (1977) hypothesis: $\pi_t \xrightarrow{+} h_{\pi,t+1} \xrightarrow{-} y_{t+1}$. As we can see from the first row of Table 4, the LR test clearly rejects the null hypotheses: $H_0 : \lambda_{\pi\pi} = \delta_{y\pi}^{(0)} = 0$. Note, that our approach still controls for the direct effect of inflation on growth ($\gamma_{y\pi}^{(2)} \neq 0$). Interestingly, the direct effect of nominal uncertainty on growth (the second stage of the Friedman hypothesis: $h_{\pi,t} \xrightarrow{-} y_t$) is in agreement with an indirect impact that works via the inflation channel.

Table 4
Likelihood ratio tests

$\pi_t \xrightarrow{+} h_{\pi,t+1} \xrightarrow{-} y_{t+1}$ (2 legs of the Friedman hypothesis)	$H_0 : \lambda_{\pi\pi} = \delta_{y\pi}^{(0)} = 0,$ $H_0 : \gamma_{y\pi}^{(2)} = 0$	10.55 [<0.01]
$h_{\pi,t} \xrightarrow{+} \pi_{t+3} \xrightarrow{-} y_{t+5}$ (Cukierman–Meltzer/Gillman–Kejak)	$H_0 : \delta_{\pi\pi}^{(3)} = \gamma_{y\pi}^{(2)} = 0$	7.92 [0.02]
$\pi_t \xrightarrow{+} h_{\pi,t+1} \xrightarrow{-} h_{y,t+2}$ (2 legs of the Dotsey–Sarte conjecture)	$H_0 : \lambda_{\pi y} = b_{y\pi} = 0$	13.68 [<0.01]
$y_t \xrightarrow{-} h_{\pi,t+1} \xrightarrow{+} h_{y,t+2}$ (Brunner conjecture/Logue–Sweeney)	$H_0 : \lambda_{\pi y} = b_{y\pi} = 0$	9.10 [0.01]
$y_t \xrightarrow{-} h_{\pi,t+1} \xrightarrow{+} h_{y,t+2} \xrightarrow{-} \pi_{t+3}$ (Brunner/Logue–Sweeney/★)	$H_0 : \lambda_{\pi y} = b_{y\pi} = \delta_{\pi y}^{(1)} = 0$	14.41 [<0.01]
$h_{\pi,t} \xrightarrow{+} \pi_{t+3} \xrightarrow{+} h_{y,t+4}$ (Cukierman–Meltzer/Dotsey–Sarte)	$H_0 : \delta_{\pi\pi}^{(3)} = \lambda_{y\pi} = 0$	4.29 [0.12]
$h_{y,t} \xrightarrow{-} \pi_{t+1} \xrightarrow{+} h_{\pi,t+2}$ (★/Ungar––Zilberfarb)	$H_0 : \delta_{\pi y}^{(1)} = \lambda_{\pi\pi} = 0$	16.50 [<0.01]
$h_{y,t} \xrightarrow{+} y_t \xrightarrow{-} h_{\pi,t+1}$ (Blackburn–Pelloni/Brunner conjecture)	$H_0 : \delta_{yy}^{(0)} = \lambda_{\pi y} = 0$	11.44 [<0.01]

Notes: The table reports the results of the likelihood ratio tests discussed in the text. The numbers in brackets are *p*-values. ⋆ : As yet, there is no theory regarding the direct effect of real uncertainty on inflation.

Higher nominal uncertainty leads to an increase in inflation, as predicted by Cukierman and Meltzer (1986), which in turn reduces output growth (the Gillman and Kejak, 2005, theory): $h_{\pi,t} \xrightarrow{+} \pi_{t+3} \xrightarrow{-} y_{t+5}$. That is, the LR test rejects the null hypothesis: $\mathrm{H}_0 : \delta_{\pi\pi}^{(3)} = \gamma_{y\pi}^{(2)} = 0$ (see the second row of Table 4).

Second, similarly to the Friedman hypothesis, the Dotsey and Sarte (2000) conjecture has two stages. Their model suggests that as average money growth rises, nominal variability increases and real growth rates become more volatile: $\pi_t \xrightarrow{+} h_{\pi,t+1} \xrightarrow{+} h_{y,t+2}$. While the first stage of the Dotsey and Sarte (2000) conjecture is identical to the first stage of the Friedman (1977) hypothesis, the second stage coincides with the Logue and Sweeney (1981) theory. An LR test confirms this hypothesis by rejecting the null hypothesis: $\mathrm{H}_0 : \lambda_{\pi\pi} = b_{y\pi} = 0$ (see the third row of Table 4). Also, note that the evidence for the Dotsey and Sarte (2000) conjecture is in line with the direct level effect of inflation on output uncertainty: $\lambda_{y\pi} > 0$.

Third, as long as we control for the asymmetric effects of the squared growth residuals on real variability, we do not find a significant direct level effect of growth on its uncertainty.[13] However, there is a negative indirect impact that works through changes in inflation uncertainty: $y_t \xrightarrow{+} h_{\pi,t+1} \xrightarrow{+} h_{y,t+2}$. That is, the null hypothesis $\mathrm{H}_0 : \lambda_{\pi y} = b_{y\pi} = 0$ is rejected (see the fourth row of Table 4). Theoretically speaking the impact is based on the interaction of two effects. A higher growth rate will reduce nominal uncertainty (the Brunner, 1993, conjecture) and, therefore, real variability (the Logue and Sweeney, 1981, theory). Hence, both inflation and growth affect real uncertainty indirectly via the nominal variability channel. Whereas the former impact is positive (as predicted by Dotsey and Sarte, 2000) the latter one is negative. This finding highlights the importance of modelling not only the in-mean effects but the level effects as well.

Fifth, although the direct effect of growth on inflation ($\gamma_{\pi y}^{(3)}$) is insignificant in Table 3, we find strong evidence for an *indirect* effect which works via the volatility channel: $y_t \xrightarrow{-} h_{\pi,t+1} \xrightarrow{+} h_{y,t+2} \xrightarrow{-} \pi_{t+3}$. That is, the LR test clearly rejects the null hypothesis: $\mathrm{H}_0 : \lambda_{\pi y} = b_{y\pi} = \delta_{\pi y}^{(1)} = 0$ (see the fifth row of Table 4).

Sixth, the positive direct effect of nominal uncertainty on real variability could also work indirectly via inflation: $h_{\pi,t} \xrightarrow{+} \pi_{t+3} \xrightarrow{+} h_{y,t+4}$. However, as we can see from the sixth row of Table 4, the null hypothesis $\mathrm{H}_0 : \delta_{\pi\pi}^{(3)} = \lambda_{y\pi} = 0$ is not rejected. Next, although there is a lack of a direct impact of real variability on nominal uncertainty, we find evidence for an indirect effect. The indirect effect works via either the inflation or growth channel. More specifically, real variability affects inflation/output growth, which then affects nominal uncertainty: $h_{y,t} \xrightarrow{-} \pi_{t+1} \xrightarrow{+} h_{\pi,t+2}$ and $h_{y,t} \xrightarrow{+} y_t \xrightarrow{-} h_{\pi,t+1}$. The latter effect is a combination of the Blackburn and Pelloni (2005) theory and the Brunner (1993) conjecture. Clearly, the LR tests reject the null hypotheses $\mathrm{H}_0 : \delta_{\pi y}^{(1)} = \lambda_{\pi\pi} = 0$ and $\mathrm{H}_0 : \delta_{yy}^{(0)} = \lambda_{\pi y} = 0$ and confirm both indirect effects

[13] Interestingly, when we ignore the asymmetry then growth has a negative impact on its variability (results not reported).

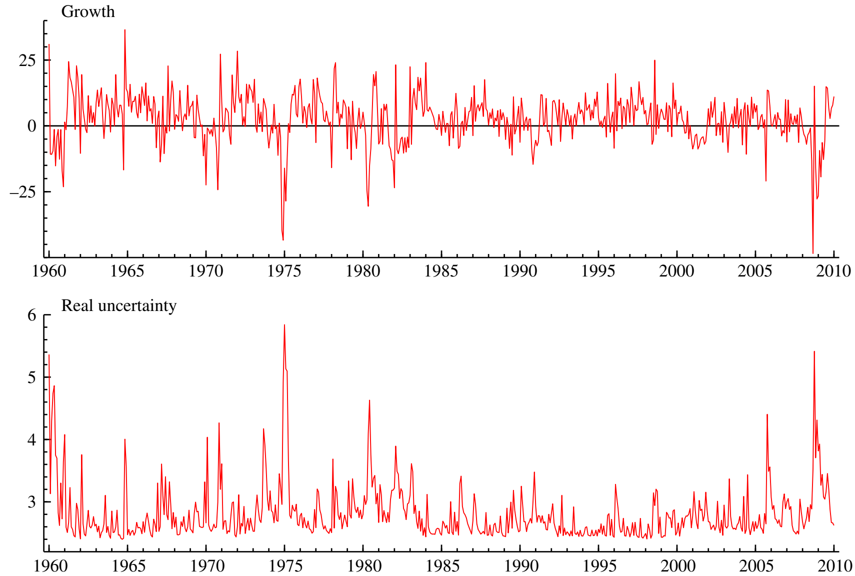

Figure 1. US output growth and real uncertainty (standard deviations) in the period 1960:01–2010:01.

(see the last two rows of Table 4). Note that the evidence in favour of the indirect effects is (partly) in line with the Fuhrer (1997) theory, which implies a trade-off between the two variabilities.

Finally, note that apart from the fact that lagged output is no longer significant in the inflation equation, the direct effect of lagged inflation on growth is also much weaker (and less significant), when we appropriately control for asymmetries and level effects. On the other hand, the cross in-mean effects become more important. Thus, the comparison between the results presented in Table 2 and 3 clearly reveals the importance of the correct modelling of the conditional variances for estimating the bidirectional effects between inflation and growth. This interpretation is strongly confirmed by an LR test, which rejects the restricted model in Table 2 in favour of the unrestricted one in Table 3 at the 1% level.

Figure 1 shows the output growth series and the real uncertainty measure as obtained from the estimation presented in Table 3. The figure clearly shows the strong relation between negative growth rates and high real uncertainty. It also shows the decline in output volatility from the 1980s onwards, i.e. the Great Moderation. Similarly, Figure 2 provides a visual impression of the inflation and nominal uncertainty series in the relevant period.

V Discussion and Comparison with Related Studies

Table 5 presents a summary of the findings in the recent literature regarding all twelve relationships between the four variables when using US data. The last row shows the results of the current study.

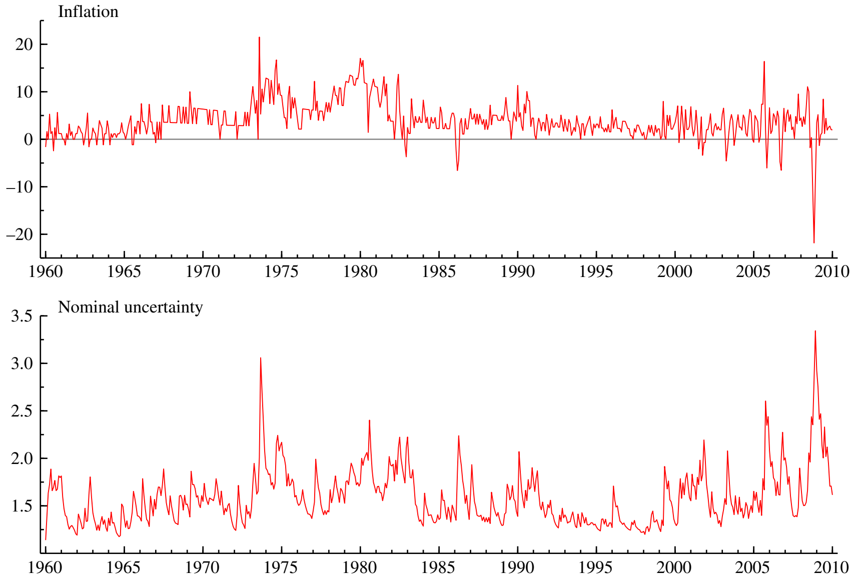

Figure 2. US inflation and nominal uncertainty (standard deviations) in the period 1960:01–2010:01.

The previous GARCH time series studies that examined the inflation-growth link in the United States use various sample periods, data frequencies and empirical methodologies. Some GARCH studies of this issue utilize the simultaneous estimation approach. For example, Baillie *et al.* (1996) and Fountas and Karanasos (2006) employ univariate GARCH models that allow for simultaneous feedback between the conditional mean and variance of inflation and growth respectively. Other recent studies have used bivariate GARCH-in-mean models – either the CCC (Grier and Perry, 2000) or the BEKK specification (Grier *et al.*, 2004, Bredin and Fountas, 2005, Shields *et al.*, 2005) – to examine the impact of macroeconomic uncertainty on performance. Some researchers employ the two-step Granger-causality approach. For example, Grier and Perry (1998), Conrad and Karanasos (2005) and Fountas and Karanasos (2007) estimate univariate GARCH models, while Karanasos and Kim (2005) and Fountas *et al.* (2006) use bivariate BEKK and CCC GARCH formulations respectively. In the first step, the estimated models are used to generate conditional variances of inflation and output growth as proxies of nominal and real uncertainty and, in the second step, Granger-causality tests are performed.

Inflation-growth link

We find a mixed bidirectional feedback between inflation and growth: $\pi \overset{-}{\rightarrow} y$, $y \overset{+}{\rightarrow} \pi$. As Table 5 shows our finding is in line with Grier *et al.* (2004), Shields *et al.* (2005) and Fountas and Karanasos (2007). However, none of these three

Table 5
The inflation-growth link in the United States. Summary of previous and present studies

Papers	Sample; Data	Cross (mean and variance) effects				In mean effects				Level effects				Signific. effects
		$\pi \to y$	$y \to \pi$	$h_\pi \to h_y$	$h_y \to h_\pi$	$h_\pi \to \pi$	$h_\pi \to y$	$h_y \to \pi$	$h_y \to y$	$\pi \to h_\pi$	$\pi \to h_y$	$y \to h_\pi$	$y \to h_y$	
In-mean-level models														
Univariate specifications														
BCT, 96		x	x	x	x	0	x	x	x	0	x	x	x	0
FK, 06	1860–1999; IPI	x	+	x	x	x	x	x	0	x	–	x	–	3
Bivariate specifications														
GP, 00	48–96; PPI, IPI	–	x	x	x	0	$-^{R_c}$	0	0	x	x	x	x	$2; 1^{R_c}$
E*, 04	66–00; CPI, IPI	–	x	x	x	x	–	x	x	x	x	x	x	2
GHOS, 04	47–00; PPI, IPI	–	+	$+^{R_+}$	$+^{R_+}$	$-^{R_c}$	$-^{R_c}$	$-^{R_c}$	$+^{R_c}$	x	x	x	x	$8; 2^{R_+}, 4^{R_c}$
BF, 05	57–03; PPI, IPI	0	0	$+^{R_+}$	$+^{R_+}$	0	$-^{R_c}$	0	$+^{R_c}$	x	x	x	x	$4; 2^{R_+}, 2^{R_c}$
SOHB, 05	47–00; PPI, IPI	–	+	$+^{R_+}$	$+^{R_+}$	$-^{R_c}$	$-^{R_c}$	$-^{R_c}$	$+^{R_c}$	x	x	x	x	$8; 2^{R_+}, 4^{R_c}$
CK, 10	60–07; CPI, IPI	x	x	+	–	x	x	x	x	x	x	x	x	2
Granger-causality tests														
Univariate specifications														
GP, 98	48–93; CPI	+	x	x	x	–	x	x	x	+	x	x	x	3
CK, 05	62–00; CPI	x	x	x	x	0	x	x	x	+	x	x	x	1
Bivariate specifications														
KK, 05	57–00; PPI, IPI	x	x	+	–	x	x	x	x	x	x	x	x	2
FKK, 06	57–00; WPI, IPI	–	0	0	0	±	–	0	+	+	0	0	–	6
FK, 07	57–00; PPI, IPI	–	+	x	x	+	–	–	0	+	x	x	x	6
Present study	60–10; CPI, IPI	–	+	+	$-^{I}$	+	–	–	+	+	+	–	$-^{I}$	12

Notes: The $\overset{x_t}{\underset{z_t}{\to}}$ column presents the effect of x_t on z_t. +/−: The effect is positive/negative. 0: The effect is zero; x: The link is not examined. *: This is a trivariate model. $^{R_+}$: The effect is restricted to being positive. ^{R_c}: The effect is restricted to being contemporaneous. ^{I}: This is an indirect effect. BCT: Baillie, Chung and Tieslau; BF: Bredin and Fountas; CK: Conrad and Karanasos; E: Elder; GHOS: Grier, Henry, Olekalns and Shields; GP: Grier and Perry; FKK: Fountas, Karanasos and Kim; FK: Fountas and Karanasos; KK: Karanasos and Kim; SOHB: Shields, Olekalns, Henry and Brooks. Overall effect: this is the sign which appears most frequently in each column.

studies have taken level effects into account. When we allow for level effects and, in particular, the negative influence of growth on inflation uncertainty (as predicted by Brunner, 1993) the direct positive impact of growth on inflation disappears. Nevertheless, we establish that the negative effect works indirectly via the volatility channel.

Volatility feedback

We find that nominal uncertainty has a positive direct impact on real variability. While we do not establish a direct effect in the opposite direction, there is evidence for a negative indirect effect of real variability on nominal uncertainty which works via either inflation or output growth. Our results are very much in line with Conrad and Karanasos (2010). In this preceding study, we had found evidence for direct volatility spillovers in both directions, whereby the spillover from real variability to nominal uncertainty was negative. However, the results in Conrad and Karanasos (2010) were based on data ending in 2007.[14] Other studies that employed bivariate GARCH-in-mean models use either a CCC or a BEKK GARCH specification, and, hence, impose either no feedback or a positive one. Only Karanasos and Kim (2005) find evidence for a mixed volatility feedback but they employ the two-step approach.

In-mean effects

According to our results, the two own in-mean effects are positive ($h_\pi \overset{+}{\rightarrow} \pi$; $h_y \overset{+}{\rightarrow} y$), whereas the two cross in-mean effects are negative ($h_\pi \overset{-}{\rightarrow} y$; $h_y \overset{-}{\rightarrow} \pi$). While the effects of nominal and real uncertainty on output growth occur contemporaneously, increasing output volatility lowers the average inflation rate with a time lag of 1 month ($h_{y,t} \overset{-}{\rightarrow} \pi_{t+1}$). Similarly, the effect of nominal uncertainty on inflation materializes with a delay of 3 months ($h_{\pi,t} \overset{+}{\rightarrow} \pi_{t+3}$). While the signs of the estimated in-mean effects are largely in line with the previous literature, it is important to highlight that previous studies that employed bivariate GARCH-in-mean models (e.g. Grier *et al.*, 2004, and Shields *et al.*, 2005) restricted the in-mean effects to being contemporaneous.[15]

Level effects

Finally, we find that higher inflation increases nominal and real uncertainty ($\pi \overset{+}{\rightarrow} h_\pi, h_y$), whereas the effect of growth is negative ($y \overset{-}{\rightarrow} h_\pi, h_y$). The latter effect of growth on real uncertainty is found to work indirectly via the volatility channel. These level effects have not been accounted for in the previous

[14] In a working paper version (see Conrad and Karanasos, 2008) of this article, we estimated a UECCC model without asymmetries and level effects for data ending in 2007. For this model and data, we also found direct volatility spillovers in both directions. The observation that the direct spillover from real variability to nominal uncertainty disappears when the sample is extended until 2010 might be explained by the fact that output growth as well as inflation became highly volatile during the recent crisis. In addition, the model used in the working paper version neither allowed for level effects nor for asymmetries.

[15] As mentioned before, in Conrad *et al.* (2010) we also allow for lagged in-mean effects but restrict the lags to be the same for both conditional variances. See also Karanasos and Zeng (2013) for a more detailed analysis of UK data.

studies which employed bivariate GARCH-in-mean models. From the studies that employ the two-step approach only Fountas *et al*. (2006) test and find, as this study does, a negative/positive effect of growth/inflation on its uncertainty. However, in Fountas *et al*. (2006) the cross-level effects are found to be insignificant.

VI Sensitivity Analysis

In this section, we analyze the robustness of our findings with respect to changes in our baseline specification.

Specification in standard deviations

As a first robustness check, we replace $\mathbf{h}_{t-r}$ by $\mathbf{h}_{t-r}^{\wedge 1/2}$ in equation (1), i.e. we express the in-mean effects in terms of standard deviations instead of conditional variances. As can be seen from Table 6, our main conclusions remain unchanged. The in-mean effects are significant ($\delta_{\pi y}$ at the 10% level, the other coefficients at the 1% or 5% level) and of the same signs as before. While the impact of inflation on real uncertainty is no longer significant, we now find a significantly negative level effect of growth on real uncertainty. This direct negative impact is in line with the indirect influence via the Brunner conjecture and the Logue-Sweeney theory discussed above.

Linear-level effects

In the estimation presented in Table 7, we replace the exponential by linear level effects. Again, our results remain unchanged, i.e. the level effect of inflation on nominal and real variability is significant and positive, while growth

Table 6
Bivariate model with standard deviations in-mean

$$\pi_t = \ldots + \underset{(0.011)}{0.012}\, y_{t-3} \ldots + \underset{(0.139)}{0.292}\sqrt{h_{\pi,t-3}} - \underset{(0.044)}{0.071}\sqrt{h_{y,t-1}} + \varepsilon_{\pi,t} \quad (12)$$

$$y_t = \ldots - \underset{(0.107)}{0.177}\, \pi_{t-2} \ldots - \underset{(0.703)}{2.498}\sqrt{h_{\pi,t}} + \underset{(0.263)}{1.090}\sqrt{h_{y,t}} + \varepsilon_{y,t} \quad (13)$$

$$\begin{aligned}\begin{pmatrix} h_{\pi,t} \\ h_{y,t} \end{pmatrix} &= \begin{pmatrix} \underset{(0.256)}{-0.224} \\ \underset{(3.595)}{28.065} \end{pmatrix} + \begin{pmatrix} \underset{(0.040)}{0.266} & - \\ \underset{(0.284)}{0.713} & \underset{(0.125)}{0.459} \end{pmatrix} \begin{pmatrix} \varepsilon^2_{\pi,t-1} \\ \varepsilon^2_{y,t-1} \end{pmatrix} \\ &+ \begin{pmatrix} - & - \\ - & -\underset{(0.129)}{0.353} \end{pmatrix} \begin{pmatrix} \mathbf{1}_{\{\varepsilon_{\pi,t-1}>0\}}\varepsilon^2_{\pi,t-1} \\ \mathbf{1}_{\{\varepsilon_{y,t-1}>0\}}\varepsilon^2_{y,t-1} \end{pmatrix} + \begin{pmatrix} \underset{(0.072)}{0.496} & - \\ \underset{(0.574)}{1.504} & - \end{pmatrix} \begin{pmatrix} h_{\pi,t-1} \\ h_{y,t-1} \end{pmatrix} \\ &+ \begin{pmatrix} \exp(\underset{(0.031)}{0.107}\pi_{t-1}) & -\underset{(0.078)}{0.187}\,\mathbf{1}_{\{y_{t-1}<0\}} y_{t-1} \\ \exp(\underset{(0.156)}{0.159}\pi_{t-1}) & \exp(-\underset{(0.077)}{0.125} y_{t-1}) \end{pmatrix} \end{aligned} \quad (14)$$

$$h_{\pi y,t} = \underset{(0.044)}{0.033}\sqrt{h_{\pi,t} h_{y,t}} \quad (15)$$

Notes: See Table 2.

Table 7
Bivariate model with linear level effects

$$\pi_t = \ldots + \underset{(0.011)}{0.010}\, y_{t-3} \ldots + \underset{(0.139)}{0.292}\, h_{\pi,t-3} - \underset{(0.0017)}{0.003}\, h_{y,t-1} + \varepsilon_{\pi,t} \tag{16}$$

$$y_t = \ldots - \underset{(0.093)}{0.265}\, \pi_{t-2} \ldots - \underset{(0.081)}{0.250}\, h_{\pi,t} + \underset{(0.009)}{0.032}\, h_{y,t} + \varepsilon_{y,t} \tag{17}$$

$$\begin{aligned}\begin{pmatrix} h_{\pi,t} \\ h_{y,t} \end{pmatrix} &= \begin{pmatrix} \underset{(0.248)}{0.491} \\ \underset{(3.660)}{26.814} \end{pmatrix} + \begin{pmatrix} \underset{(0.041)}{0.275} & - \\ \underset{(0.280)}{0.718} & \underset{(0.121)}{0.473} \end{pmatrix} \begin{pmatrix} \varepsilon^2_{\pi,t-1} \\ \varepsilon^2_{y,t-1} \end{pmatrix} \\ &\quad + \begin{pmatrix} - & - \\ - & -\underset{(0.127)}{0.321} \end{pmatrix} \begin{pmatrix} \mathbf{1}_{\{\varepsilon_{\pi,t-1} > 0\}} \varepsilon^2_{\pi,t-1} \\ \mathbf{1}_{\{\varepsilon_{y,t-1} > 0\}} \varepsilon^2_{y,t-1} \end{pmatrix} + \begin{pmatrix} \underset{(0.071)}{0.512} & - \\ \underset{(0.576)}{0.881} & - \end{pmatrix} \begin{pmatrix} h_{\pi,t-1} \\ h_{y,t-1} \end{pmatrix} \\ &\quad + \begin{pmatrix} \underset{(0.076)}{0.200}\, \pi_{t-1} & -\underset{(0.081)}{0.184}\, \mathbf{1}_{\{y_{t-1} < 0\}} y_{t-1} \\ \underset{(0.971)}{1.981}\, \pi_{t-1} & - \end{pmatrix}\end{aligned} \tag{18}$$

$$h_{\pi y,t} = \underset{(0.043)}{0.032} \sqrt{h_{\pi,t} h_{y,t}} \tag{19}$$

Notes: See Table 2.

has a negative impact on nominal uncertainty. The sign and the significance of all four in-mean effects is as before.

Lagged level effects

We also investigate the robustness of our findings with respect to the lag order of the level variables (results not reported). Recall that in the baseline specification, we employ only the first lag of inflation and output growth as explanatory variables in the conditional variances. However, we find the level effects of higher order lags to be either of the same sign as before or insignificant. In particular, the negative effect of growth on nominal uncertainty is confirmed at lag two for negative as well as positive growth rates. (Recall that in Table 3 only negative growth rates have a significant level effect on inflation uncertainty.)

Sub-periods

To control for possible changes in the conduct of monetary policy, we re-estimate our favoured specification by interacting the main variables of interest with dummy variables for the period 1980–2010 (results not reported). Alternatively, one could also determine structural breaks endogenously. While our conclusions regarding the link between the two variabilities remain unchanged, we find some changes in the in-mean effects. Among the four interaction terms, only those on the own in-mean effects are significant, that is, the one on nominal uncertainty in the inflation equation and the one on real variability in the growth equation. Both own in-mean effects tail off from the 1980s onwards. The fact that the positive effect of nominal uncertainty on inflation becomes weaker is in line with the observation that the Fed became more inflation focused during that time and, hence, supports the Holland

Table 8
AR-UECCC-GARCH(1,1)-in-mean model for quarterly data

$$\pi_t = \ldots + \underset{(0.020)}{0.032}\, y_{t-1} \ldots + \underset{(0.200)}{0.457}\; h_{\pi,t} - \underset{(0.007)}{0.012}\, h_{y,t} + \varepsilon_{\pi,t} \quad (20)$$

$$y_t = \ldots - \underset{(0.116)}{0.318}\, \pi_{t-1} \ldots - \underset{(0.241)}{0.540}\, h_{\pi,t} + \underset{(0.063)}{0.016}\, h_{y,t} + \varepsilon_{y,t} \quad (21)$$

$$\Lambda = \begin{pmatrix} \underset{(0.023)}{0.052} & - \\ \underset{(0.041)}{0.153} & - \end{pmatrix} \quad (22)$$

Notes: See Table 2.

argument (see also Grier and Perry, 2000). A damped positive effect of real variability on growth is expected from the literature on the Great Moderation, i.e. the observation that the volatility of growth has considerably declined since the early 1980's.[16]

GDP growth instead of industrial production

As a final robustness check, we re-estimate our model using quarterly GDP growth data (see Table 8). Although the quarterly frequency reduces the number of observations, checking whether our results are robust to using GDP growth instead of industrial production is important, in particular, since the share of GDP that is due to industrial production has considerably decreased over the last decades. Again, we find that inflation has a direct and highly significant negative effect on output growth. On the other hand, the direct effect of growth on inflation is positive but significant at the 12% level only. Three of the four in-mean effects are significant. Inflation uncertainty increases inflation while it affects growth negatively (both significant at the five percent level). Real uncertainty reduces inflation (significant at the 12% level), but has no significant influence on growth. All four in-mean effects have the same signs as for the monthly data, but are now contemporaneous. Similarly, the level effects of inflation on the two uncertainties are positive and significant. Although there is no evidence for volatility interaction in the quarterly data, we still find evidence for strong asymmetry in real uncertainty ($\widehat{g}_{yy} = -0.255(0.118)$).

VII Conclusions

We have employed an augmented version of the UECCC GARCH model to investigate the relationship among inflation, nominal uncertainty, output growth and real variability using US data. The main advantage of this new specification is that it allows for (1) in-mean effects which can occur at different lags, (2) level effects specified in an exponential fashion, (3) asymmetries in the conditional variances and (4) volatility feedback of either sign. Thus,

[16] The recent studies by McConnell and Perez-Quiros (2000) and Stock and Watson (2002) highlight the importance of the reduction in US GDP growth volatility in the last two decades and its implications for growth theory.

we have been able to test the economic theories which imply causal relationships between the four variables in a unified framework. Our results highlight the importance of modelling all possible interactions simultaneously. In particular, we find that many effects work indirectly via the uncertainty channel. For example, we find strong support for the two stages of the Friedman (1977) hypothesis, that is higher inflation increases nominal uncertainty, which then negatively affects output growth. Maybe even more importantly, we show that the positive direct effect of output growth on inflation disappears, once we appropriately model the asymmetries and level effects. On the other hand, the indirect effect via real variability becomes stronger. Interestingly, in all cases the signs of the direct and the indirect effects are the same. Thus, our results suggest that the methodologies employed in previous studies – which exclusively focused on the direct effects – have masked the existence of the potentially even more important indirect effects.

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