The Three-Dimensional Hyperbolic Asymmetric Power HEAVY model: the importance of Range-based Volatility and Structural Breaks.

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Abstract

This paper proposes the three-dimensional HEAVY system of daily, intra-daily and range-based volatility equations. We augment the bivariate model with a third volatility metric, the Garman-Klass estimator, and enrich the trivariate system with power transformations, asymmetries, and long memory. All three power transformed conditional variances are found to be significantly affected by the powers of squared returns, realized measure, and range-based volatility as well. Other findings are as follows. First, hyperbolic memory fits the realized measure better, whereas fractional integration is more suitable for the power transformed returns and Garman-Klass volatility. Second, the augmentation of the HEAVY framework with the range-based volatility estimator further improves its forecasting accuracy. Third, the structural breaks applied to the trivariate system capture the time-varying behavior of the parameters, in particular during the global financial crisis. Finally, our results reveal interesting insights for investments, market risk measurement, and policymaking.

Keywords: asymmetries, financial crisis, HEAVY model, high-frequency data, hyperbolic long memory, power transformations, realized volatility, structural breaks.

JEL classification: C22, C58, G01, G15

1 Introduction

Financial volatility lies at the core of empirical finance research, with direct employment in investments, risk management practices, and financial stability oversight. Reliable modeling and accurate forecasting of the volatility pattern has been the main objective of financial applications for business operations, given that volatility is considered as one of the fundamental input variables in estimations and decision processes of any corporation on derivatives pricing, portfolio immunization, investment diversification, firm valuation, and funding choices. Financial volatility is also closely inspected by policymakers since it entails critical destabilizing threats for the financial system.

We develop a three-dimensional HEAVY[1] model by augmenting the bivariate system of Shephard and Sheppard (2010) with a third variable, namely, the range-based Garman-Klass volatility. Another contribution is the enrichment of the trivariate model with asymmetries and power transformations through the APARCH structure of Ding et al. (1993). Motivated by the established merits of this framework, which considerably improves Bollerslev's (1986) GARCH process by adding leverage and power effects (see, for example, Brooks et al., 2000, Karanasos and Kim, 2006), we similarly extend the trivariate system with these two features to explore its superiority over the benchmark specification. One of our key findings is that each of the three powered conditional variances is significantly affected by the first lags of all three power transformed variables, that is, squared negative returns, realized variance and Garman-Klass volatility.

We also include long memory (either fractionally integrated or hyperbolic), by employing the framework of Davidson (2004) (see, Schoffer, 2003 and Dark, 2005, 2010, as well). We find that a fractionally integrated specification better fits the squared returns and the Garman-Klass volatility, whereas a hyperbolic type of memory is preferred for the realized measure. The long memory feature reinforces our main argument that the lagged values of the power transformations of all three aforementioned variables move the dynamics of the three powered conditional variances. The fractionally integrated (asymmetric power) model for the returns and the Garman-Klass volatility equations pools information across both low-frequency and high-frequency based volatility indicators. Similarly, the more richly parametrized hyperbolic process for the realized variance equation is bolstered with low-frequency information as well since the lagged value of the powered squared negative returns improves the forecasting performance of the model. We examine the various specifications in depth by investigating their performance over five stock indices. Finally, in the presence of structural breaks, which are apparent in the three power transformed volatility measures, we re-estimate the trivariate system including dummy variables, and we present the time-varying behavior of the parameters. Focusing on the recent global financial crisis, we

[1] The acronym HEAVY is derived by 'High-frEquency-bAsed VolatilitY' in Shephard and Sheppard (2010).

observe that their values consistently increase after the crisis break.

Following the burst of the 2008 crisis, when volatilities rose sharply and persistently with crucial systemic risk externalities, we witnessed a resurgence of regulators' and academics' interest in meaningful volatility estimates, while, at the same time, practitioners remained alert to improving the relevant volatility frameworks on a day-to-day basis. Financial economics scholars focused on volatility as a potent catalyst of systemic risk build-up, which policymakers tried to limit. We demarcate this study from the extant finance bibliography by extending the benchmark HEAVY model with asymmetries, power transformations, long memory and Garman-Klass volatility providing a well-defined framework that adequately fits the volatility process. We further demonstrate the forecasting superiority of the proposed model over the benchmark specification using a rolling window out-of-sample forecasting procedure. The three-dimensional system of volatility equations, we establish, is ready-to-use, not only on stock market returns but also on further asset classes or financial instruments (exchange rate, cryptocurrency, commodity, real estate, and bond returns) and multiple financial economics applications of business operations, such as bonds investing, foreign exchange trading and commodities hedging, core daily functions in the treasuries of most financial and non-financial corporations.

Overall, our proposed volatility modeling framework improves the HEAVY model, with important implications for market practitioners and policymakers on forecasting the trajectory of the financial returns' second moment. Volatility modeling and forecasting are essential for asset allocation, pricing, and risk hedging strategies. A reliable volatility forecast, exploiting in full the high-frequency domain, is the input variable of paramount importance for the processes of derivatives pricing, effective cross-hedging, Value-at-Risk measurement, investment allocation and portfolio optimization with different asset classes and financial instruments. Moreover, the robust volatility modeling approach we introduce provides a useful tool not only for market players but also for policymakers. Policymaking includes continuous oversight duties and prudential regulation practices. In this vein, it is imperative for the authorities to account for the volatility of financial markets across every aspect of the financial system's policy responses, both post-crisis through stabilization policy reactions and pre-crisis through proactive assessment of financial risks. The hyperbolic asymmetric power HEAVY framework we propose here has been shown to perform significantly better than the benchmark specification both in the short- and the long-term forecasting horizons. Trading and risk management processes mostly use one- to ten-day forecasts while policymakers are involved in longer-term predictions of financial volatility. Hence, we illustrate our model's forecasting superiority with a Value-at-Risk example that provides both risk management and policy implications.

The remainder of the paper is structured as follows. In Section 2 we detail the three-dimensional HEAVY formulation and our first extension, which allows for asymmetries and power transformations.

Section 3 describes the data and Section 4 presents the results for the asymmetric power specification. The next Section lays out the long memory process and discusses the relevant empirical findings. In Section 6, we calculate multiple-step-ahead forecasts to measure the out-of-sample performance of the various specifications. The following Section takes into consideration the presence of structural breaks. Finally, Section 8 concludes the analysis.

2 The HEAVY Framework

There are several studies introducing non-parametric estimators of realized volatility using high-frequency market data. Andersen and Bollerslev (1998), Andersen et al. (2001) and Barndorff-Nielsen and Shephard (2002) were the first that econometrically formalized the realized variance with quadratic variation-like measures, while Barndorff-Nielsen et al. (2008, 2009) focused on the realized kernel estimation as a realized measure which is more robust to noise.

A large body of empirical research focuses on modeling and forecasting the realized volatility. Various studies combine it with the conditional variance of returns. Engle (2002b) proposed the GARCH-X process, where the former is included as an exogenous variable in the equation of the latter. Corsi et al. (2008) suggested the HAR-GARCH formulation for modeling the volatility of realized volatility. Hansen et al. (2012) introduced the Realized GARCH model that corresponds more closely to the HEAVY framework of Shephard and Sheppard (2010), which jointly estimates conditional variances based on both daily (squared returns) and intra-daily (it uses the realized measure - kernel and variance - as a measure of ex-post volatility) data, so that the system of equations adopts to information arrival more rapidly than the classic daily GARCH process. One of its advantages is the robustness to certain forms of structural breaks, especially during the crisis periods, since the mean reversion and short-run momentum effects result in higher quality performance in volatility level shifts and more reliable forecasts. Borovkova and Mahakena (2015) employed a HEAVY specification with a skewed-t error distribution, while Huang et al. (2016) incorporated the HAR structure of the realized measure in the GARCH conditional variance specification in order to capture the long memory of the volatility dynamics.

The benchmark HEAVY model of Shephard and Sheppard (2010) can be extended in many directions. We allow for power transformations, leverage effects and long memory (see Section 5 below) in the conditional variance process. We run the estimated benchmark specification of Shephard and Sheppard (2010), enriched with the three key features to improve volatility modeling and forecasting further.

2.1 Benchmark Model

The HEAVY model uses two variables: the close-to-close stock returns (r_t) and the realized measure of variation based on high-frequency data, RM_t. We first form the signed square rooted (SSR) realized measure as follows: $\widetilde{RM}_t = sign(r_t)\sqrt{RM_t}$, where $sign(r_t) = 1$, if $r_t \geqslant 0$ and $sign(r_t) = -1$, if $r_t < 0$.

In this paper we test the inclusion of an alternative measure of volatility to the HEAVY framework, that is we employ the classic range-based estimator of Garman and Klass (1980), hereafter GK. We further form the SSR GK volatility ($\widetilde{GK}_t = sign(r_t)\sqrt{GK_t}$).

We assume that the returns, the SSR realized measure and GK volatility are characterized by the following relations:

$$r_t = e_{rt}\sigma_{rt},\ \widetilde{RM}_t = e_{Rt}\sigma_{Rt},\ \widetilde{GK}_t = e_{gt}\sigma_{gt}, \tag{1}$$

where the stochastic term e_{it} is independent and identically distributed ($i.i.d$), $i = r, R, g$; σ_{it} is positive with probability one for all t and it is a measurable function of $\mathcal{F}_{t-1}^{(XF)}$, that is the filtration generated by all available information through time $t-1$. We will use $\mathcal{F}_{t-1}^{(HF)}$ ($X = H$) for the high-frequency past data, i.e., for the case of the realized measure, or $\mathcal{F}_{t-1}^{(LoF)}$ ($X = Lo$) for the low-frequency past data, i.e., for the case of the close-to-close returns. Hereafter, for notational convenience, we will drop the superscript XF.

In the HEAVY/GARCH model e_{it} has zero mean and unit variance. Therefore, the three series have zero conditional means, and their conditional variances are given by

$$\mathbb{E}(r_t^2\,|\mathcal{F}_{t-1}) = \sigma_{rt}^2,\ \mathbb{E}(\widetilde{RM}_t^2\,|\mathcal{F}_{t-1}) = \mathbb{E}(RM_t\,|\mathcal{F}_{t-1}) = \sigma_{Rt}^2, \text{ and } \mathbb{E}(\widetilde{GK}_t^2\,|\mathcal{F}_{t-1}) = \mathbb{E}(GK_t\,|\mathcal{F}_{t-1}) = \sigma_{gt}^2, \tag{2}$$

where $\mathbb{E}(\cdot)$ denotes the expectation operator. The three equations are called HEAVY-i, $i = r, R, g$ for the returns, the realized measure, and Garman Klass volatility, respectively.

2.2 Asymmetric Power Formulation

The asymmetric power (AP) specification for the three-dimensional (3D) HEAVY$(1,1)$ consists of the following equations (in what follows for notational simplicity, we will drop the order of the model if it is $(1,1)$):

$$(1-\beta_i L)(\sigma_{it}^2)^{\frac{\delta_i}{2}} = \omega_i + (\alpha_{ir} + \gamma_{ir}s_{t-1})L(r_t^2)^{\frac{\delta_r}{2}} + (\alpha_{iR} + \gamma_{iR}s_{t-1})L(RM_t)^{\frac{\delta_R}{2}} + (\alpha_{ig} + \gamma_{ig}s_{t-1})L(GK_t)^{\frac{\delta_g}{2}}, \tag{3}$$

where L is the lag operator, $\delta_i \in \mathbb{R}_{>0}$ (the set of the positive real numbers) are the power parameters, for $i = r, R, g$, and $s_t = 0.5[1 - sign(r_t)]$, that is, $s_t = 1$ if $r_t < 0$ and 0 otherwise; γ_{ii}, γ_{ij} ($i \neq j$) are the own and cross leverage parameters, respectively[2]; positive γ_{ii}, γ_{ij} means larger contribution of negative

[2] This type of asymmetry was introduced by Glosten et. al. (1993).

'shocks' in the volatility process (in our long memory AP specification we will replace $\alpha_{ii} + \gamma_{ii}s_{t-1}$ by $\alpha_{ii}(1 + \gamma_{ii}s_{t-1})$; see Section 5 below, and, in particular, eq. (4)). In this specification the powered conditional variance, $(\sigma_{it}^2)^{\delta_i/2}$, is a linear function of the lagged values of the power transformed squared returns, realized measure and GK volatility.

We will distinguish between three different asymmetric cases: the double one (DA: $\gamma_{ij} \neq 0$ for all i and j) and two more, own asymmetry (OA: $\gamma_{ij} = 0$ for $i \neq j$ only) and cross asymmetry (CA: $\gamma_{ii} = 0$).

The α_{iR} and γ_{iR} are called the (six) Heavy parameters (own when $i = R$ and cross when $i \neq R$). These parameters capture the impact of the realized measure on the three conditional variances. Similarly, the α_{ir} and γ_{ir} (six in total) are called the Arch parameters (own when $i = r$ and cross for $i \neq r$). They depict the influence of the squared returns on the three conditional variances. Finally, the α_{ig} and γ_{ig} are called the (six) Garman parameters. These parameters capture the effects of the GK volatility on the three conditional variances.

The asymmetric power model is equivalent to a trivariate AP-GARCH process for the returns, the SSR realized measure, and GK volatility (see, for example, Conrad and Karanasos, 2010). If all twelve Arch and Garman parameters are zero, then we have the AP version of the benchmark HEAVY specification where the only unconditional regressor is the first lag of the powered RM_t. Finally, we should mention that all the parameters in this trivariate system should take non-negative values (see, for example, Conrad and Karanasos, 2010).

To sum up, the bivariate benchmark model (eq. (2)) of Shephard and Sheppard (2010)[3] is characterized by two conditional variance equations, the GARCH(1,0)-X formulation for returns and the GARCH(1,1) formulation for the SSR realized measure:

$$\text{HEAVY-}r\text{: } (1 - \beta_r L)\sigma_{rt}^2 = \omega_r + \alpha_{rR}L(RM_t),$$

$$\text{HEAVY-}R\text{: } (1 - \beta_R L)\sigma_{Rt}^2 = \omega_R + \alpha_{RR}L(RM_t).$$

Eq. (3) gives the general formulation of our asymmetric power extension, which adds asymmetries, power transformations and the GK volatility to the benchmark specification (see the supplementary Appendix B for our theoretical considerations). We also use the existing Gaussian quasi-maximum likelihood estimators (QMLE) and multistep-ahead predictors already applied in the APARCH framework (see, for example, He and Teräsvirta, 1999, Laurent, 2004, Karanasos and Kim, 2006). We will first estimate the three conditional variance equations in the general form with all Heavy, Arch, Garman, and Asymmetry parameters given by eq. (3) and in case a parameter is insignificant, we will exclude it and this will result

[3]The benchmark HEAVY specification as established by Shephard and Sheppard (2010) does not incorporate our third variable, that is GK volatility.

in a reduced form that is statistically preferred for each volatility process.

3 Data Description

The HEAVY framework is estimated for five stock indices' returns, realized and GK volatilities. According to the analysis in Shephard and Sheppard (2010), this formulation improves the volatility modeling considerably by allowing momentum and mean reversion effects and adjusting quickly to the structural breaks in volatility. We extend the bivariate model to a trivariate system by including the GK volatility measure as an additional variable. We also extend the benchmark specification in Shephard and Sheppard (2010), by adding the features of power transformed conditional variances, leverage effects, and long memory (see Section 5 below) in the volatility process. Moreover, in order to identify the possible recent global financial crisis effects on the volatility process and to take into account the structural breaks in the three powered series (squared returns, realized measure and GK volatility), in Section 7 we incorporate dummies in our empirical investigation. The analysis with the structural breaks can be considered as an alternative to the long memory investigation.

We use daily data for five stock market indices extracted from the Oxford-Man Institute's (OMI) realized library version 0.3 of Heber et al. (2009): Dow Jones Industrial Average from the US (DJ), Korea Composite Stock Price Index from South Korea (KOSPI), CAC 40 from France (CAC), All Ordinaries from Australia (AORD), and MXSE IPC from Mexico (IPC). Our sample covers the period from 03/01/2000 to 30/09/2019 for most indices. The OMI's realized library includes daily stock market returns and several realized volatility measures calculated on high-frequency data from the Reuters DataScope Tick History database. The data are first cleaned and then used in the realized measures calculations. According to the library's documentation, the data cleaning consists of deleting records outside the time interval that the stock exchange is open. Some minor manual changes are also needed when results are ineligible due to the rebasing of indices. We use the daily closing prices, P_t^C, to form the daily returns as follows: $r_t = \ln(P_t^C) - \ln(P_{t-1}^C)$, and two realized measures as drawn from the library: the realized kernel and the 5-minute realized variance. The estimation results using the two alternative measures are very similar, so we present only the ones with the realized variance (the results for the realized kernel are available upon request).

3.1 Realized Measures

The library's realized measures are calculated in the way described in Shephard and Sheppard (2010). The realized kernel, which we use as an alternative to the realized variance (results are not reported but they are available upon request), is calculated using a Parzen weight function as follows: $RK_t =$

$\sum_{k=-H}^{H} k(h/(H+1))\gamma_h$, where $k(x)$ is the Parzen kernel function with $\gamma_h = \sum_{j=|h|+1}^{n} x_{j,t} x_{j-|h|,t}$; $x_{jt} = X_{t_{j,t}} - X_{t_{j-1,t}}$ are the 5-minute intra-daily returns where $X_{t_{j,t}}$ are the intra-daily log-prices and $t_{j,t}$ are the times of trades on the t-th day. Shephard and Sheppard (2010) declared that they selected the bandwidth of H as in Barndorff-Nielsen et al. (2009).

The 5-minute realized variance, RV_t, which we choose to present here, is calculated with the formula: $RV_t = \sum x_{j,t}^2$. Heber et al. (2009) additionally implement a subsampling procedure from the data to the most feasible level in order to eliminate the stock market noise effects. The subsampling involves averaging across many realized variance estimations from different data subsets (see also the references in Shephard and Sheppard, 2010 for realized measures surveys', noise effects and subsampling procedures).

3.2 GK Volatility

Using data on the daily high, low, opening and closing prices of each index in the OMI's realized library we generate an additional daily measure of price volatility. To avoid the microstructure biases introduced by high-frequency data and based on the conclusion of Chen et al. (2006), that the range-based and high-frequency integrated volatility provide essentially equivalent results, we construct the daily GK volatility as follows:

$$GK_t = \frac{1}{2}u_t^2 - (2\ln 2 - 1)c_t^2,$$

where u_t and c_t are the differences in the natural logarithms (as of time t) of the high and low and of the closing and opening prices, respectively. The Garman-Klass is an open-to-close range-based volatility estimator that is documented as a more precise volatility proxy, with superior empirical performance in the GARCH framework. Recently, Molnár (2016) has demonstrated that the inclusion of the Parkinson and GK estimators in the Range-GARCH model he proposed, outperforms the standard GARCH$(1, 1)$, and it performs particularly better in situations, where volatility level changes rapidly. Several studies have also discussed the improvement of the GARCH framework through the open-to-close range-based volatility proxies, regarded as more accurate than the close-to-close squared returns: they exclude the noise from the dynamics of the opening jumps and they ensure greater accuracy in volatility forecasting through the range information they provide (see Chou et al. 2010, 2015, Molnár, 2012 and the references therein). Therefore, we incorporate the GK variable in our HEAVY system, in order to improve the model's forecasting performance.

Table 1 presents the five stock indices extracted from the database and provides volatility estimations for each one's squared returns, realized variances, and GK volatilities time series for the respective sample period (see also the DJ series graphs in the supplementary Appendix A, Figures A.1-A.4). We calculate the standard deviation of the series and the annualized volatility. Annualized volatility is the square rooted mean of 252 times the squared return or the realized variance. The standard deviations are always lower than the annualized volatilities. The realized variances and the GK volatilities have lower annualized volatilities and standard deviations than the squared returns since they ignore the overnight effects and are affected by less noise. The returns represent the close-to-close yield, the realized variance the open-to-close variation and the GK volatility the open-to-close range-based variation. The annualized volatility of the realized and GK measure is between 10% and 18%, while the squared returns show figures from 14% to 24%.

Table 1. Data Description

	Sample period			r_t^2		RV_t		GK_t	
Index	Start date	End date	Obs.	Avol	sd	Avol	sd	Avol	sd
DJ	03/01/2000	27/09/2019	4950	0.178	0.040	0.165	0.026	0.145	0.022
KOSPI	04/01/2000	30/09/2019	4857	0.235	0.067	0.174	0.022	0.170	0.027
CAC	03/01/2000	30/09/2019	5034	0.222	0.052	0.182	0.022	0.175	0.021
AORD	04/01/2000	30/09/2019	4985	0.143	0.022	0.108	0.008	0.100	0.009
IPC	03/01/2000	30/09/2019	4953	0.202	0.044	0.144	0.018	0.155	0.017

Notes: Avol is the annualized volatility and sd is the standard deviation.

Next, we examine the sample autocorrelations of the power transformed absolute returns $|r_t|^{\delta_r}$, signed square rooted realized variance $|SSR\_RM_t|^{\delta_R}$, and GK volatility $|SSR\_GK_t|^{\delta_g}$ for various values of δ_i. Figures 1, 2, and 3 show the autocorrelograms of the Dow Jones index from lag 1 to 120 for $\delta_r = 1.3, 1.7, 2.0$, $\delta_R = 1.1, 1.5, 2.0$, and $\delta_g = 1.0, 1.5, 2.0$ (similar autocorrelograms for the other four indices available upon request). The sample autocorrelations for $|r_t|^{1.3}$ are greater than the sample autocorrelations of $|r_t|^{\delta_r}$ for $\delta_r = 1.7, 2.0$ at every lag up to at least 120 lags. In other words, the most interesting finding from the autocorrelogram is that $|r_t|^{\delta_r}$ has the strongest and slowest decaying autocorrelation when $\delta_r = 1.3$. Similarly, for the realized measure and GK volatility, the powers with the strongest autocorrelation function are $\delta_R = 1.1$ and $\delta_g = 1.0$, respectively. Furthermore, Figures 4, 5, and 6 present the sample autocorrelations of $|r_t|^{\delta_r}$, $|SSR\_RM_t|^{\delta_R}$, and $|SSR\_GK_t|^{\delta_g}$ as a function of δ_i for lags $1, 12, 36, 72$ and 96. For example, for lag 12, the highest autocorrelation values of power transformed absolute returns and signed square rooted realized and GK volatility are calculated closer to the power of 1.5 and 1.0, respectively. These figures explain our motivation to extend the Benchmark HEAVY through the APARCH framework of Ding et al. (1993) and confirm the power choice of our econometric models, which is $\delta_r = 1.3$ for returns, $\delta_R = 1.1$ for the realized measure, and $\delta_g = 1.0$ for GK volatility (see Section 4).

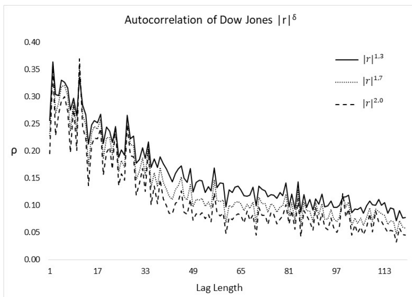

Figure 1. Autocorrelation of Dow Jones $|r_t|^{\delta_r}$ for $\delta_r = 1.3, 1.7, 2.0$

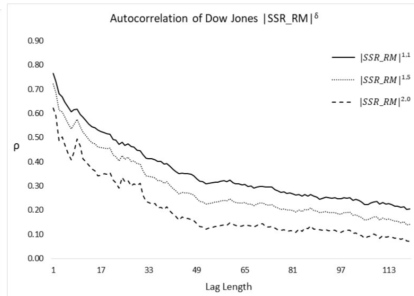

Figure 2. Autocorrelation of Dow Jones $|SSR\_RM_t|^{\delta_R}$ for $\delta_R = 1.1, 1.5, 2.0$

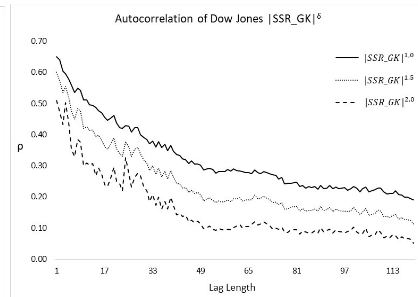

Figure 3. Autocorrelation of Dow Jones $|SSR\_GK_t|^{\delta_g}$ for $\delta_g = 1.0, 1.5, 2.0$

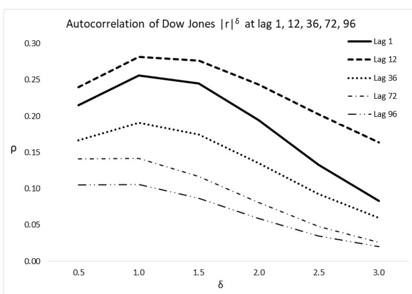

Figure 4. Autocorrelation of Dow Jones $|r_t|^{\delta_r}$ at lags $1, 12, 36, 72, 96$

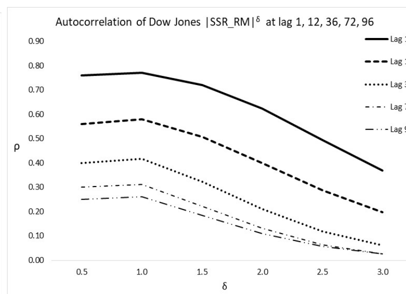

Figure 5. Autocorrelation of Dow Jones $|SSR\_RM_t|^{\delta_R}$ at lags $1, 12, 36, 72, 96$

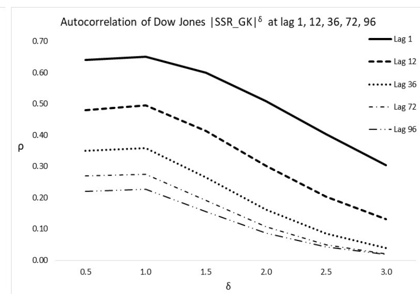

Figure 6. Autocorrelation of Dow Jones $|SSR\_GK_t|^{\delta_g}$ at lags $1, 12, 36, 72, 96$

4 Estimated Models

Building upon the introduction of the GARCH-X process by Engle (2002b) to include realized measures as exogenous regressors in the conditional variance equation, Han and Kristensen (2014) and Han (2015) studied the asymptotic properties of this new specification with a fractionally integrated (nonstationary) process included as covariate (see also Francq and Thieu, 2019). Moreover, Nakatani and Teräsvirta (2009) and Pedersen (2017) focused on the multivariate case, the so-called extended constant conditional correlation, which allows for volatility spillovers and they developed inference and testing for the QMLE parameters (see also Ling and McAleer, 2003 for the asymptotic theory of vector ARMA-GARCH processes). For the extended HEAVY models, we employ the existing Gaussian QMLE and multistep-ahead predictors applied in the APARCH framework (see, for example, He and Teräsvirta 1999, Laurent, 2004, Karanasos and Kim, 2006). Following Pedersen and Rahbek (2019), we first test for arch effects and after rejecting the conditional homoscedasticity hypothesis we apply one-sided significance tests of the covariates added to the estimated GARCH processes.

We first estimate the bivariate benchmark formulation as in Shephard and Sheppard (2010), that is, without asymmetries and power transformations, obtaining very similar results (Table 2). For the benchmark specification, the only unconditional regressor in both equations is the first lag of the RM_t. In other words, the chosen returns equation is a GARCH$(1,0)$-X process leaving out the own Arch effect, α_{rr}, from lagged squared returns since it becomes insignificant when we add the cross effect of the lagged realized measure as regressor, with a Heavy coefficient, α_{rR}, high in value and significance across all indices. The momentum parameter, β_r, is estimated around 0.44 to 0.84. For the SSR realized variance, the best-chosen model is the GARCH$(1,1)$ without the cross effect from lagged squared returns. The Heavy term, α_{RR}, is estimated between 0.25 and 0.47 and the momentum, β_R, is around 0.53 to 0.74. The benchmark system of equations chosen (three alternative GARCH models are tested for each dependent variable with order: $(1,1)$, $(1,0)$-X, and the most general one, that is, $(1,1)$-X) is the same as in Shephard and Sheppard (2010) with similar parameter values and the identical conclusion that the realized measure of variation does all the work of moving around the conditional variances of stock returns and the SSR realized variance. The benchmark's conclusion, as we show in this study, does not hold for the more richly parametrized asymmetric power model. More importantly, according to the Sign Bias test (SBT) of Engle and Ng (1993), the asymmetric effect is obviously omitted from the benchmark specification with the sign coefficient always significant (p-values lower than 0.02).

Table 2. The Benchmark HEAVY model

	DJ	KOSPI	CAC	AORD	IPC
	Panel A: Stock Returns, HEAVY-r				
	$(1-\beta_r L)\sigma^2_{rt} = \omega_r + \alpha_{rR} L(RM_t)$				
β_r	0.65 (15.99)***	0.67 (10.58)***	0.44 (7.68)***	0.78 (26.61)***	0.84 (28.45)***
α_{rR}	0.39 (7.62)***	0.62 (5.27)***	0.82 (9.05)***	0.37 (6.88)***	0.25 (5.17)***
Q_{12}	15.43 [0.22]	12.94 [0.37]	12.05 [0.44]	14.40 [0.28]	15.40 [0.21]
SBT	3.07 [0.00]	2.32 [0.02]	2.29 [0.02]	2.60 [0.01]	4.91 [0.00]
lnL	−6336.82	−7599.64	−7762.45	−5728.74	−7582.94
	Panel B: Realized Measure, HEAVY-R				
	$(1-\beta_R L)\sigma^2_{Rt} = \omega_R + \alpha_{RR} L(RM_t)$				
β_R	0.57 (14.06)***	0.53 (13.11)***	0.57 (17.08)***	0.74 (30.57)***	0.67 (11.56)***
α_{RR}	0.44 (9.26)***	0.47 (10.59)***	0.42 (12.40)***	0.25 (10.45)***	0.33 (5.19)***
Q_{12}	12.52 [0.41]	16.20 [0.18]	9.54 [0.66]	16.77 [0.16]	16.23 [0.17]
SBT	3.68 [0.00]	3.49 [0.00]	2.25 [0.02]	2.47 [0.01]	2.99 [0.00]
lnL	−5930.41	−6140.66	−6819.26	−4362.39	−5823.11

Notes: The numbers in parentheses are t-statistics. ***, **, * denote significance at the 0.01, 0.05, 0.10 level, respectively. Q_{12} is the Box-Pierce Q-statistics on the standardized residuals with 12 lags. SBT denotes the Sign Bias test of Engle and Ng (1993). lnL denotes the log-likelihood value for each specification. The numbers in square brackets are p-values.

Moving to our proposed extension of the benchmark bivariate system, Tables 3A-3C present the estimation results for the chosen three-dimensional asymmetric power specifications (see also the 3D-Benchmark model in Appendix A, Table A.1). Wald and t-tests are used to test the significance of the Heavy, Arch, and Garman parameters, rejecting the null hypothesis at 10% in all cases. We should highlight the fact that since all the parameters take non-negative values, we use one-sided tests (see, for example, Pedersen and Rahbek, 2019).

For all three dependent variables, we statistically prefer the double asymmetric power (DAP) specification since most power transformed conditional variances are significantly affected by own and cross asymmetries. KOSPI's realized measure equation is the only case where we prefer the cross asymmetric power (CAP) model since own asymmetries are insignificant and therefore excluded. Furthermore, we estimate the power terms separately with a two-stage procedure, as follows: We, first, estimate univariate asymmetric power specifications for the returns, the realized measure, and GK volatility. The Wald tests for the estimated power terms (available upon request) reject the hypothesis of $\delta_i = 2$ in all cases. In the second stage, we use the estimated powers, δ_r, δ_R, and δ_g, from the first step to power transform each series' conditional variance and incorporate them into the trivariate model. The sequential procedure produces the fixed power term values, which are the same for the three specifications (δ_r, δ_R, and δ_g are common for Panels A, B, and C).

For the returns, the estimated power, δ_r, is between 1.30 and 1.60 (see Table 3A). The Heavy asymmetry parameter, γ_{rR}, is significant and around 0.06 (min. value) to 0.13 (max. value). Although α_{rr} is insignificant and excluded in all cases, the own asymmetry parameter is significant with $\gamma_{rr} \in [0.08, 0.11]$. In addition, the cross Garman parameter, α_{rg}, is significant and $0.07 \leq \alpha_{rg} \leq 0.13$ in all cases. In other words, the lagged values of all three powered variables, that is, the negative signed realized measure, the squared negative returns, and the GK volatility, drive the model of the power transformed conditional variance of returns. Moreover, the momentum parameter, β_r, is estimated to be around 0.80 to 0.90. Obviously, all five indices generated very similar DAP specifications.

Table 3A. The 3D-DAP-HEAVY model

	DJ	KOSPI	CAC	AORD	IPC
	Panel A: Stock Returns				
	$(1-\beta_r L)(\sigma_{rt}^2)^{\frac{\delta_r}{2}} = \omega_r + \gamma_{rr} s_{t-1} L(r_t^2)^{\frac{\delta_r}{2}} + \gamma_{rR} s_{t-1} L(RM_t)^{\frac{\delta_R}{2}} + \alpha_{rg} L(GK_t)^{\frac{\delta_g}{2}}$				
β_r	0.81 (45.11)***	0.82 (25.25)***	0.80 (24.33)***	0.87 (55.05)***	0.91 (65.59)***
α_{rg}	0.10 (4.78)***	0.13 (4.66)***	0.12 (3.19)***	0.08 (3.83)***	0.07 (3.99)***
γ_{rr}	0.08 (5.08)***	0.09 (4.84)***	0.10 (6.00)***	0.09 (6.46)***	0.11 (8.39)***
γ_{rR}	0.10 (4.76)***	0.12 (3.48)***	0.13 (4.32)***	0.07 (2.76)***	0.06 (3.90)***
δ_r	1.30	1.50	1.40	1.60	1.60
δ_R	1.10	1.20	1.10	1.30	1.00
δ_g	1.00	1.20	1.10	1.20	1.20
Q_{12}	15.89 [0.20]	11.64 [0.48]	15.12 [0.24]	13.73 [0.19]	8.12 [0.62]
SBT	1.16 [0.24]	0.84 [0.40]	0.31 [0.75]	0.41 [0.68]	0.11 [0.91]
lnL	−5974.12	−6933.25	−7078.02	−5584.51	−6890.68

Notes: See notes in Table 2.

Similarly, for the realized measure the most preferred specification is the DAP one in most cases, as the estimated power is $\delta_R \in [1.00, 1.30]$ (see Table 3B). Both Heavy parameters, α_{RR} and γ_{RR}, are mostly significant: α_{RR} is around 0.05 (min. value) to 0.27 (max. value), while γ_{RR}, is between 0.03 and 0.05. Only for the KOSPI index, the own asymmetries are insignificant and excluded. Moreover, the cross Arch asymmetry parameter is significant with $\gamma_{Rr} \in [0.04, 0.09]$, as well as the cross Garman parameter, α_{Rg}, (with estimated values between 0.05 and 0.12). This means that the power transformed conditional variance of $\widetilde{RM}_t$ is significantly affected by the lagged values of all three powered variables: squared negative returns, realized measure, and GK volatility. Lastly, the momentum parameter, β_R, is estimated to be around 0.62 to 0.81.

Table 3B. The 3D-DAP-HEAVY model

	DJ	KOSPI	CAC	AORD	IPC
		Panel B: Realized Measure			
		$(1-\beta_R L)(\sigma^2_{Rt})^{\frac{\delta_R}{2}} = \omega_R +$ $(\alpha_{RR} + \gamma_{RR} s_{t-1}) L (RM_t)^{\frac{\delta_R}{2}} +$ $\gamma_{Rr} s_{t-1} L (r_t^2)^{\frac{\delta_r}{2}} + \alpha_{Rg} L (GK_t)^{\frac{\delta_g}{2}}$			
β_R	0.71 (41.14)***	0.62 (25.07)***	0.72 (36.11)***	0.81 (47.29)***	0.73 (31.23)***
α_{RR}	0.10 (5.62)***	0.27 (12.04)***	0.16 (7.57)***	0.05 (3.43)***	0.19 (9.48)***
α_{Rg}	0.12 (7.94)***	0.06 (3.83)***	0.05 (4.38)***	0.08 (5.96)***	0.05 (4.33)***
γ_{RR}	0.05 (5.09)***		0.03 (3.61)***	0.04 (4.60)***	0.03 (2.82)***
γ_{Rr}	0.08 (8.23)***	0.04 (9.44)***	0.05 (11.25)***	0.04 (6.75)***	0.09 (5.74)***
δ_R	1.10	1.20	1.10	1.30	1.00
δ_r	1.30	1.50	1.40	1.60	1.60
δ_g	1.00	1.20	1.10	1.20	1.20
Q_{12}	15.18 [0.23]	14.40 [0.28]	15.16 [0.23]	13.72 [0.19]	13.68 [0.20]
SBT	0.64 [0.52]	0.71 [0.48]	0.74 [0.46]	1.01 [0.31]	1.12 [0.26]
lnL	-5264.81	-5346.23	-5865.78	-4151.74	-5230.93

Notes: See notes in Table 2.

Finally, regarding the GK volatility the DAP specification is again the chosen one (see Table 3C). In particular, the own power term is $1.00 \leq \delta_g \leq 1.20$ in all cases. In addition, the Heavy (α_{gR}), the own asymmetry, γ_{gg}, and the Arch asymmetry, γ_{gr}, parameters are significant in all cases. In other words, the first lags of all three powered variables (realized measure, negative signed GK volatility, and squared negative returns) drive the model of the power transformed conditional variance of $\widetilde{GK}_t$.

Table 3C. The 3D-DAP-HEAVY model

	DJ	KOSPI	CAC	AORD	IPC
	Panel C: GK volatility				
	$(1-\beta_g L)(\sigma_{gt}^2)^{\frac{\delta_g}{2}} = \omega_g + \gamma_{gg} L(GK_t)^{\frac{\delta_g}{2}} + \alpha_{gR} s_{t-1} L(RM_t)^{\frac{\delta_R}{2}} + \gamma_{gr} s_{t-1} L(r_t^2)^{\frac{\delta_r}{2}}$				
β_g	0.76 (35.34)***	0.65 (18.49)***	0.75 (28.51)***	0.82 (44.50)***	0.84 (42.41)***
α_{gR}	0.11 (8.07)***	0.26 (9.18)***	0.16 (7.44)***	0.09 (7.93)***	0.09 (5.57)***
γ_{gg}	0.07 (7.87)***	0.02 (1.77)*	0.05 (5.82)***	0.03 (3.52)***	0.03 (3.11)***
γ_{gr}	0.05 (6.62)***	0.05 (7.85)***	0.04 (8.33)***	0.04 (6.83)***	0.05 (8.66)***
δ_g	1.00	1.20	1.10	1.20	1.20
δ_r	1.30	1.50	1.40	1.60	1.60
δ_R	1.10	1.20	1.10	1.30	1.00
Q_{12}	13.70 [0.32]	14.98 [0.24]	15.04 [0.24]	13.75 [0.30]	13.72 [0.31]
SBT	0.78 [0.44]	0.91 [0.36]	1.16 [0.25]	0.90 [0.37]	1.08 [0.28]
lnL	−4990.36	−5213.30	−5677.71	−3421.67	−5838.98

Notes: See notes in Table 2.

Overall, our results show strong Heavy effects (captured by the γ_{rR}, α_{RR}, γ_{RR} and α_{gR} parameters), asymmetric Arch influences (as the estimated γ_{rr}, γ_{Rr} and γ_{gr} are significant), as well as Garman impacts (captured by the α_{rg}, α_{Rg} and γ_{gg} parameters). According to the log-likelihood (lnL) values reported, the log-likelihood is always higher for the DAP specifications compared to the benchmark ones, that is without asymmetries and powers, proving the superiority of our model's in-sample estimation. The SBT statistics further show that the asymmetric effect is not omitted any more since the sign coefficients are insignificant, with p-values consistently higher than 0.24.

5 Long Memory Extension

5.1 Hyperbolic Formulation

In this Section, we extend the 3D-DAP-HEAVY framework by incorporating long memory. First, we present the most general hyperbolic (HY) specification (see, for example, in the context of a univariate

GARCH model Davidson, 2004, Dark, 2005, 2010, and Schoffer, 2003):

$$\begin{aligned}(1-\beta_r L)[(\sigma_{rt}^2)^{\frac{\delta_r}{2}}-\omega_r] &= A_r(L)(1+\gamma_{rr}s_t)(r_t^2)^{\frac{\delta_r}{2}}+(\alpha_{rR}+\gamma_{rR}s_{t-1})L(RM_t)^{\frac{\delta_R}{2}}+(\alpha_{rg}+\gamma_{rg}s_{t-1})L(GK_t)^{\frac{\delta_g}{2}},\\ (1-\beta_R L)[(\sigma_{Rt}^2)^{\frac{\delta_R}{2}}-\omega_R] &= A_R(L)(1+\gamma_{RR}s_t)(RM_t)^{\frac{\delta_R}{2}}+(\alpha_{Rr}+\gamma_{Rr}s_{t-1})L(r_t^2)^{\frac{\delta_r}{2}}+(\alpha_{Rg}+\gamma_{Rg}s_{t-1})L(GK_t)^{\frac{\delta_g}{2}},\\ (1-\beta_g L)[(\sigma_{gt}^2)^{\frac{\delta_g}{2}}-\omega_g] &= A_g(L)(1+\gamma_{gg}s_t)(GK_t)^{\frac{\delta_g}{2}}+(\alpha_{gR}+\gamma_{gR}s_{t-1})L(RM_t)^{\frac{\delta_R}{2}}+(\alpha_{gr}+\gamma_{gr}s_{t-1})L(r_t^2)^{\frac{\delta_r}{2}},\end{aligned} \tag{4}$$

with

$$A_i(L)=(1-\beta_i L)-(1-\phi_i L)[(1-\zeta_i)+\zeta_i(1-L)^{d_i}],\ i=r,R,g,$$

where $|\phi_i|<1$, d_i, is the fractional differencing parameter with $0\leq d_i\leq 1$, and ζ_i, is the amplitude or hyperbolic parameter with $0\leq\zeta_i\leq 1$. In other words, we have three long memory parameters, ϕ_i, ζ_i, and d_i. So, now the Heavy parameters are nine in total. Similarly, the Arch parameters are nine, and the Garman parameters as well.

If $\zeta_i=0$ and $\phi_i-\beta_i=\alpha_{ii}$, the HYDAP specification reduces to the DAP ones (see eq. (3)), since in this case $A_i(L)=\alpha_{ii}L$.

The HY specification also nests the fractional integrated (FI) one (see, for example, Baillie et al., 1996, Tse, 1998, Karanasos et al., 2004, and Conrad and Karanasos, 2006) by imposing the restriction $\zeta_i=1$. In this case $A_i(L)$, in eq. (4) becomes

$$A_i(L)=(1-\beta_i L)-(1-\phi_i L)(1-L)^{d_i}. \tag{5}$$

Finally, note that the sufficient conditions of Dark (2005, 2010) for the non-negativity of the conditional variance of a HYAPARCH $(1,d_i,1)$ specification are: $\omega_i>0$, $\beta_i-\zeta_i d_i\leq\phi_i\leq\frac{2-d_i}{3}$ and $\zeta_i d_i(\phi_i-\frac{1-d_i}{2})\leq\beta_i(\phi_i-\beta_i+\zeta_i d_i)$, $i=r,R,g$ (see also Conrad, 2010). When $\zeta_i=1$ they reduce to the ones for the FIGARCH $(1,d_i,1)$ model (see Bollerslev and Mikkelsen, 1996).

5.2 Long Memory Estimation Results

We further extend the HEAVY framework by incorporating long memory. For the returns and the GK volatility, the chosen specification is the FIDAP, whereas for the realized measure we select the HYDAP one (with the exception of KOSPI realized variance, where the HYCAP model is preferred). In all cases, the power terms are presented as fixed parameters since they are estimated separately using univariate models. Tables 4A-4C present the 3D-HYDAP-HEAVY results.

In the FIDAP specification for the returns (see Table 4A), d_r is close to 0.50 (around 0.38 to 0.45). In all cases, the Wald tests (available upon request) reject the null hypotheses of $d_r=0$ or 1. The other two long memory parameters, ϕ_r and the hyperbolic one, ζ_r, were insignificant and, therefore, they were excluded. The own and Garman asymmetry parameters are significant with estimated values $\gamma_{rr} \in [0.33, 0.50]$ and $\gamma_{rg} \in [0.15, 0.19]$, respectively. The Heavy parameter, α_{rR}, is significant as well and with estimated values between 0.06 and 0.12. In other words, the lagged values of all three powered variables (squared negative returns, realized measure and negative signed GK volatility) drive the model of the power transformed conditional variance of returns.

Table 4A. The 3D-HYDAP-HEAVY model

	DJ	KOSPI	CAC	AORD	IPC
	Panel A: Stock Returns, FIDAP Specification				
	$(1-\beta_r L)[(\sigma_{rt}^2)^{\frac{\delta_r}{2}} - \omega_r] = \left[(1-\beta_r L) - (1-L)^{d_r}\right](1+\gamma_{rr}s_t)(r_t^2)^{\frac{\delta_r}{2}} + \alpha_{rR}L(RM_t)^{\frac{\delta_R}{2}} + \gamma_{rg}s_{t-1}L(GK_t)^{\frac{\delta_g}{2}}$				
β_r	0.38 (4.56)***	0.41 (5.83)***	0.37 (2.74)***	0.36 (4.78)***	0.32 (2.85)***
d_r	0.42 (2.75)***	0.45 (9.23)***	0.42 (5.25)***	0.40 (7.68)***	0.38 (7.68)***
α_{rR}	0.07 (4.91)***	0.12 (2.25)**	0.10 (2.99)***	0.07 (3.12)***	0.06 (5.01)***
γ_{rr}	0.50 (5.04)***	0.33 (5.00)***	0.48 (6.11)***	0.43 (7.24)***	0.35 (5.78)***
γ_{rg}	0.17 (3.43)***	0.19 (3.27)***	0.18 (3.26)***	0.15 (2.92)***	0.16 (2.95)***
δ_r	1.30	1.50	1.40	1.60	1.60
δ_R	1.10	1.20	1.10	1.30	1.00
δ_g	1.00	1.20	1.10	1.20	1.20
Q_{12}	14.80 [0.25]	12.49 [0.41]	12.56 [0.40]	18.19 [0.11]	12.58 [0.39]
SBT	1.09 [0.28]	0.21 [0.84]	0.26 [0.79]	0.72 [0.47]	0.85 [0.40]
lnL	−5337.06	−6584.66	−6785.68	−5277.31	−6330.71

Notes: See notes in Table 2.

In the HYDAP specification for the realized measure (see Table 4B), there is strong evidence of hyperbolic memory as not only d_R but also ζ_R is significant, with estimated values $0.47 - 0.55$ and $0.66 - 0.90$, respectively, with the Wald tests (available upon request) always rejecting the null of either a FIDAP ($H_0 : \zeta_R = 1$) or a DAP formulation ($H_0 : \zeta_R = 0$). The own and the cross (Arch) asymmetric parameters, $\gamma_{RR} \in [0.18, 0.57]$ and $\gamma_{Rr} \in [0.06, 0.10]$, are also significant, as well as the Garman parameter, $\alpha_{Rg} \in [0.06, 0.15]$. Own asymmetries are insignificant and excluded in the Korean index only, where we statistically prefer a HYCAP specification. This means that the power transformed conditional variance of $\widetilde{RM}_t$ is significantly affected by the lagged values of all three powered variables: realized measure, GK volatility and squared negative returns.

Table 4B. The 3D-HYDAP-HEAVY model

	DJ	KOSPI	CAC	AORD	IPC
	Panel B: Realized Measure, HYDAP Specification				

$$(1-\beta_R L)[(\sigma^2_{Rt})^{\frac{\delta_R}{2}} - \omega_R] =$$
$$\gamma_{Rr} s_{t-1} L(r_t^2)^{\frac{\delta_r}{2}} + \alpha_{Rg} L(GK_t)^{\frac{\delta_g}{2}} +$$
$$(1-\phi_R L)[(1-\zeta_R) + \zeta_R(1-L)^{d_R}](1+\gamma_{RR} s_t)(RM_t)^{\frac{\delta_R}{2}}$$

	DJ	KOSPI	CAC	AORD	IPC
β_R	0.63 (15.77)***	0.36 (6.28)***	0.58 (14.25)***	0.44 (4.54)***	0.36 (2.33)**
ϕ_{RR}	0.33 (3.90)***	0.18 (2.13)**	0.32 (8.09)***	0.03 (7.56)***	0.05 (5.82)***
ζ_R	0.70 (17.30)***	0.66 (7.75)***	0.84 (37.40)***	0.81 (28.05)***	0.90 (14.23)***
d_R	0.53 (10.02)***	0.47 (12.10)***	0.54 (21.91)***	0.55 (9.97)***	0.47 (11.31)***
α_{Rg}	0.15 (8.56)***	0.10 (5.57)***	0.06 (3.99)***	0.12 (5.75)***	0.08 (1.88)*
γ_{RR}	0.42 (2.64)***		0.18 (3.36)***	0.57 (4.87)***	0.21 (2.99)***
γ_{Rr}	0.10 (8.80)***	0.06 (11.46)***	0.07 (11.33)***	0.07 (6.78)***	0.06 (3.61)***
δ_R	1.10	1.20	1.10	1.30	1.00
δ_r	1.30	1.50	1.40	1.60	1.60
δ_g	1.00	1.20	1.10	1.20	1.20
Q_{12}	14.97 [0.24]	14.61 [0.26]	15.55 [0.21]	14.95 [0.25]	13.50 [0.30]
SBT	0.30 [0.76]	1.03 [0.30]	0.37 [0.71]	1.05 [0.29]	0.82 [0.41]
lnL	−4261.79	−4341.16	−4853.86	−3550.86	−4767.10

Notes: See notes in Table 2.

Similarly to the model for the returns, in the FIDAP specification for the GK volatility (see Table 4C), d_g is around 0.40 to 0.44, whereas the hyperbolic parameter was insignificant. The own (Garman) and the cross Heavy asymmetric parameters, $\gamma_{gg} \in [0.10, 0.22]$ and $\gamma_{gR} \in [0.06, 0.10]$, are also significant. However, the Arch asymmetric effect, γ_{gr}, was insignificant and excluded, with the direct effect from powered squared returns, α_{gr}, included. Therefore, the lagged values of all three powered variables, that is, the squared returns and the negative signed realized variance and GK volatility, drive the model of the power transformed conditional variance of the range-based measure.

Table 4C. The 3D-HYDAP-HEAVY model

	DJ	KOSPI	CAC	AORD	IPC
	Panel C: GK volatility, FIDAP Specification				
	$(1-\beta_g L)[(\sigma_{gt}^2)^{\frac{\delta_g}{2}} - \omega_g] =$ $\alpha_{gr} L(r_t^2)^{\frac{\delta_r}{2}} + \gamma_{gR} s_{t-1}(RM_t)^{\frac{\delta_R}{2}} +$ $\left[(1-\beta_g L)-(1-L)^{d_g}\right](1+\gamma_{gg} s_t)(GK_t)^{\frac{\delta_g}{2}}$				
β_g	0.27 (3.49)***	0.26 (1.90)*	0.26 (1.93)**	0.34 (2.14)**	0.28 (4.76)***
d_g	0.41 (9.56)***	0.43 (16.27)***	0.40 (14.31)***	0.40 (7.72)***	0.44 (2.01)**
α_{gr}	0.02 (1.71)*	0.02 (2.26)**	0.02 (2.77)***	0.03 (3.20)***	0.05 (1.90)**
γ_{gg}	0.22 (4.96)***	0.15 (5.14)***	0.17 (3.46)***	0.12 (2.66)***	0.10 (2.84)***
γ_{gR}	0.10 (3.59)***	0.07 (2.68)***	0.10 (4.66)***	0.09 (3.38)***	0.06 (5.01)***
δ_g	1.00	1.20	1.10	1.20	1.20
δ_r	1.30	1.50	1.40	1.60	1.60
δ_R	1.10	1.20	1.10	1.30	1.00
Q_{12}	10.93 [0.54]	15.21 [0.23]	11.06 [0.52]	10.99 [0.53]	10.89 [0.56]
SBT	0.40 [0.69]	0.39 [0.70]	1.29 [0.20]	1.20 [0.23]	1.11 [0.27]
lnL	-4343.66	-5061.74	-5218.49	-3078.81	-5283.32

Notes: See notes in Table 2.

All in all, our long memory extension of the asymmetric power specification demonstrates once more that all powered conditional variances receive the notable impact from the first lags of the three power transformed variables. Intriguingly, this result stands in sharp contrast to the benchmark HEAVY model, where the intra-daily realized measure is not affected by squared daily returns and the daily returns conditional variance is only determined by the lagged realized measure and the lagged returns variance since the asymmetries from negative returns are completely neglected. Furthermore, the powers are estimated with the two-stage procedure same as the asymmetric power specification with similar δ_r, δ_R, and δ_g values common across the three volatility equations.

Lastly, we estimated the trivariate system with the various conditional correlation models: the CCC-Constant Conditional Correlations (Bollerslev, 1990), the DCC-Dynamic Conditional Correlations (Engle, 2002a), the ADCC-Asymmetric Dynamic Conditional Correlations (Cappiello et al., 2006) and the DECO-Dynamic Equicorrelations (Engle and Kelly, 2012). All correlation models estimate the average conditional correlations for the three volatility measures around 0.75 to 0.95. The conditional correlations extension provides identical results for the conditional variance equations (since this is a two-step approach), estimates similar correlation levels for all indices, and, most importantly, does not improve further the 3D-HYDAP-HEAVY formulation (see also supplamentary Appendix B, Section B.4). Therefore, we do not report the results (available upon request).

6 Forecast Evaluation

Following the estimation of all possible extensions to the HEAVY framework of equations, we perform multistep-ahead out-of-sample forecasting in order to compare the forecasting accuracy of the enriched specifications proposed in this study with the benchmark model introduced by Shephard and Sheppard (2010). We compute 1-, 5-, 10-, and 22-step-ahead forecasts of the (power transformed) conditional variances for the benchmark, the 3D-DAP, and the 3D-HYDAP models. We apply a rolling window in-sample estimation using 2500 observations (the initial in-sample estimation period for DJ spans from 3/1/2000 until 24/12/2009). Each model is re-estimated daily based on the 2500-day rolling sample. The resulted out-of-sample forecasts of each specification calculated for DJ are as follows: 2450 one-step-ahead, 2446 five-step-ahead, 2441 ten-step-ahead, and 2439 twenty-two-step-ahead forecasted variances. We then use the time series of the forecasted values to compute the Mean Square Error (MSE) and the QLIKE Loss Function (Patton, 2011) of each point forecast compared to the respective actual value. For each formulation and each forecast horizon, we calculate the average MSE and QLIKE to build the ratio of the forecast losses for each extended HEAVY specification to the loss of the benchmark one. A ratio lower than the unity indicates the forecasting superiority of the proposed models relative to the benchmark one. The lowest ratio means lowest forecast losses, that is the model with the best forecasting performance.

We apply the optimal predictor $|\mathbf{r}_t|^{\wedge\delta}$ (under Proposition 3 of the supplementary Appendix B) and calculate the out-of-sample forecasts. The results, presented in Table 5 for the DJ index (similar forecasting results for the other four indices available upon request), clearly show the preference for our extensions over the benchmark models across all time horizons. Regarding the returns equations (see Panel A), the FIDAP is the best performing specification in the one-day out-of-sample forecasts, while for the one-week up to the one-month forecasts the DAP specification dominates the long memory and benchmark models with the lowest MSE and QLIKE. In the realized measure equation, we get the best one-step-ahead forecasting performance from the three-dimensional system with the DAP specification (see Panel B). For the remaining time horizons, the preferred specification is the Hyperbolic one. Finally, for the GK volatility equation (Panel C), we prefer the long memory model across all time horizons apart from the five-day interval, where the 3D-DAP outperforms the Benchmark and FIDAP specifications.

Overall, the extensions proposed in our study perform significantly better than the benchmark HEAVY models in the short- and the long-term horizons, with the computed forecasts significantly closer to the actual values for the enriched HEAVY formulations. Investors, traders and risk managers can benefit from the superior short-term forecasts for one up to ten days, while policymakers should focus on the longer-term forecasting performance to predict 'safely' the one-month-forward financial volatility given the significant range-based effects.

Table 5. Mean Square Error (MSE) and QLIKE of m-step-ahead out-of-sample forecasts for DJ as a Ratio of the benchmark model.

	MSE				QLIKE			
Specifications↓ m-steps →	1	5	10	22	1	5	10	22
Panel A: Stock Returns (HEAVY-r)								
Benchmark (bivariate)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3D-DAP	0.769	**0.791**	**0.824**	**0.872**	0.711	**0.747**	**0.761**	**0.833**
3D-FIDAP	**0.741**	0.822	0.839	0.913	**0.696**	0.776	0.789	0.851
Panel B: Realized Measure (HEAVY-R)								
Benchmark (bivariate)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3D-DAP	**0.784**	0.836	0.845	0.946	**0.721**	0.744	0.780	0.865
3D-HYDAP	0.801	**0.819**	**0.837**	**0.920**	0.756	**0.684**	**0.709**	**0.786**
Panel C: GK volatility (HEAVY-g)								
Benchmark$^{\oplus}$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3D-DAP	0.804	**0.773**	0.850	0.912	0.832	**0.741**	0.841	0.897
3D-FIDAP	**0.759**	0.791	**0.821**	**0.886**	**0.794**	0.815	**0.830**	**0.868**

Notes: Bold numbers indicate minimum values across the different specifications.

$^{\oplus}$The Benchmark Heavy-$g$specification is defined in Table A.1, Panel C (Trivariate Benchmark)

The forecasting performance of the proposed models can be further examined in a real-world risk management empirical example. Value-at-Risk (VaR) is a daily metric for market risk measurement, defined as the potential loss in the value of a portfolio, over a pre-defined holding period, for a given confidence level. The most important input in the VaR calculation is the one-day volatility forecast of the risk factor relevant to the trading portfolio under scope. We directly apply our conditional variance forecasts in a long portfolio position to one Dow Jones Industrial Average index contract starting from 7/5/2019. We calculate 100 daily VaR values from 8/5/2019 to 27/9/2019 using the one-day conditional variance forecasts of each model for returns and realized measure (6 models in total). Given that the conditional mean return is zero and the returns follow the normal distribution, we, first, calculate the one-day VaR with 99% and 95% confidence level. According to the parametric approach to VaR calculation, we multiply the daily portfolio value with the one-day-ahead conditional volatility forecast (equal to the square root of the conditional variance forecast) and the left quantile at the respective confidence level of the normal distribution (the z-scores for 99% and 95% confidence level are 2.326 and 1.645, respectively). Secondly, we calculate the daily realized return of the portfolio (gains and losses) and, thirdly, we perform the backtesting exercise, comparing the realized returns with the respective one-day VaR for the 99% and 95% confidence levels. In the cases where the realized loss exceeds the respective day's VaR value, we record it as an exception in the backtesting procedure, meaning that the VaR metric fails to cover the loss of the specific day's portfolio value.

According to the backtesting results (see Table 6: Backtesting results, No. of Exceptions), the number of exceptions across all models is in line with the selected confidence level (the 99% and 95% confidence levels allow for 1 and 5 exceptions, respectively, every 100 days) and low enough to prevent supervisors from increasing the capital charges (in which case we refer to a bank's trading portfolio). The higher number of exceptions means higher market risk capital requirements for financial institutions since regulators heavily penalize banks' internal models that fail to cover trading losses through the VaR estimates. Following the Basel traffic light approach, the market risk capital charge increases when the backtesting exceptions are more than 4 in a sample of 250 daily observations and 99% confidence level. Since all models provide adequate coverage of the realized losses, we should further compare the average and minimum VaR estimates calculated based on the forecasts of each specification (Table 6: Descriptive statistics). The VaR estimate that provides the higher loss coverage with the lower capital charges is the one with the lower minimum and higher mean values. This is achieved by the realized measure specifications, where we prefer the asymmetric power models, augmented with the range-based volatility impact and enriched or not with the long memory feature. Given that the market risk capital requirement is calculated on the trading portfolio total 99% VaR (absolute value, 60-day average) adjusted by the penalty of the backtesting exceptions (higher than 4 in the 250-day sample), the bank needs the smallest possible VaR average with the larger minimum estimate in absolute terms. Thereupon, our proposed models clearly satisfy both criteria, contributing to the risk manager's VaR calculation of the volatility forecasts that better capture the loss distribution (higher extreme loss coverage with higher absolute minimum value) without inflating the capital charges (lower absolute mean).

Table 6. VaR Backtesting results and Descriptive statistics for the DJ portfolio.

	Backtesting results		Descriptive statistics			
	No. of Exceptions		99% VaR		95% VaR	
Specifications	99% VaR	95% VaR	Mean	Min.	Mean	Min.
Panel A: Stock Returns (HEAVY-r)						
Benchmark (bivariate)	1	3	-700.04	$-1,418.87$	-494.97	$-1,003.22$
3D-DAP	1	3	-656.75	$-1,346.29$	-468.80	-951.90
3D-FIDAP	1	3	-648.99	$-1,285.31$	-459.71	-923.86
Panel B: Realized Measure (HEAVY-R)						
Benchmark (bivariate)	1	3	-632.24	-934.48	-447.03	-660.72
3D-DAP	1	3	$\mathbf{-641.20}$	$\mathbf{-1,241.32}$	-456.90	-877.68
3D-HYDAP	1	3	$\mathbf{-646.83}$	$\mathbf{-1,266.41}$	-462.13	-903.79

Notes: Mean and Min. denote the average and minimum VaR estimate, respectively. Bold numbers indicate the preferred specifications for the lower market risk capital charge with the higher loss coverage.

Furthermore, the volatility forecasts produced by the 3D-HYDAP-HEAVY model are directly applicable to a wide range of business finance operations, alongside the well-established risk management practice outlined in the VaR empirical exercise. Portfolio managers should rely on the proposed framework to predict future volatility in asset allocation and minimum-variance portfolio selection complying with their clients' risk appetite. Risk averse investors' mandates specify low volatility boundaries on their portfolio positions, while risk lovers allow for higher volatilities on the risk-return trade-off of their investments. Accurate volatility predictions can also be used in a forward-looking performance evaluation context, through the risk-adjusted metrics, i.e. the Sharpe or the Treynor risk-adjusted return ratios. Traders and risk managers focus on the volatility trajectory in derivatives pricing, volatility targeting strategies and several other trading decisions. Trading and hedging in financial markets depend on risk factors whose predicted volatilities are the main input of any pricing function applied. Moreover, financial chiefs consider volatility forecasts when they decide on investment projects or funding choices (bond and equity valuation defining the cost of capital) given that expected future cash-flow variation is a critical factor in business analytics.

Finally, policymakers and authorities supervising and regulating the financial system should take into account reliable volatility forecasts in designing macro- and micro-prudential policy responses. The risk management of the financial system is structured as follows: i) identification of risk sources (both endogenous - financial market volatility - and exogenous - the macroeconomy), ii) assessment of the nature of risk factors, iii) risk measurement (micro-prudential metrics at the financial institution level and macro-prudential metrics at the system and markets level), and iv) risk mitigation with proactive regulation and crisis preparedness plans and strategies. Therefore, regulators should employ the range-informed financial volatility forecasts of the 3D-HYDAP-HEAVY model across the whole risk management process and the financial stability oversight tools, such as the early warning systems, the macro stress-tests on financial institutions and the bank capital and risk frameworks. For example, the macro stress-test scenario inputs, which include, among others, stock market volatility predictions for the financial institutions' trading books, should consider range-informed volatility estimates. Furthermore, complying with the capital and risk frameworks set by supervisors (Basel committee and central banks), financial institutions measure their trading portfolio's market risk (beyond the credit risk of their loan portfolio) with the daily Value-at-Risk (VaR) metric. Given that reliable volatility forecasts, provided by our superior modeling framework, improve the VaR estimates considerably, supervisors should encourage banks to improve their market risk internal models with more accurate range-informed volatility forecasts based on both low- and high-frequency data.

7 Structural Breaks

Since we analyzed the superiority of our asymmetric power extensions for the HEAVY system, in this Section, we investigate the impact of structural changes (detected in the three power transformed time series used as dependent variables) on the Heavy, Arch and Garman estimated parameters. The time-varying behavior of these parameters can be significant around a financial crisis break, in particular, indicative of the crisis effects on the volatility pattern. As an alternative to the long memory specification, we incorporate structural break dummies in the 3D-DAP-HEAVY system. We first identify the structural breaks in the three volatility series for DJ, focusing mainly on the recent global financial crisis, and study their impact on the three-dimensional framework. The methodology in Bai and Perron (1998, 2003a,b) is employed to test for structural breaks. They address the problem of testing for multiple structural changes in a least squares context and under very general conditions on the data and the errors. In addition to testing for the presence of breaks, these statistics identify the number and location of multiple breaks. So, we identify the structural breaks in the three powered series (power transformations [PT] of squared returns, realized measure, and GK volatility, see Tables 3A-3C) with the Bai and Perron methodology (see Table 7 and Figures 7-9). We use the breaks identified in order to build the slope dummies for the various parameters. One break date for the recent financial crisis of 2007/08 is detected so that we can focus on the crisis effect. We also detect one break date before and one after the crisis.

Table 7. The break dates for Dow Jones

	1^{st} Break	2^{nd} Break	3^{rd} Break
r	28/04/2003	31/10/2007	30/11/2011
R	**06/08/2003**	**30/10/2007**	**20/12/2011**
g	06/08/2003	31/10/2007	20/12/2011

Notes: Bai & Perron breaks identification: Results selected from the repartition procedure for 1% significance level with 5 maximum number of breaks and 0.15 trimming parameter. Dates in bold indicate that the corresponding dummy coefficient is used in the 3D-DAP-HEAVY model.

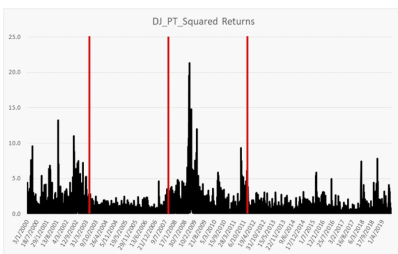

Figure 7. Dow Jones PT Squared Returns with Breaks

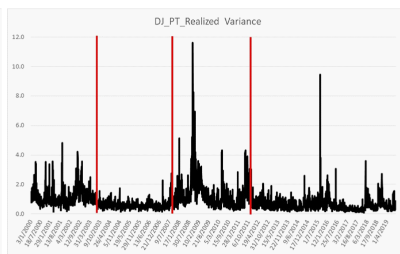

Figure 8. Dow Jones PT Realized Variance with Breaks

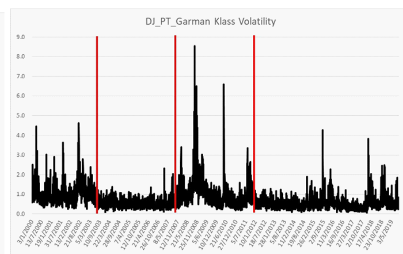

Figure 9. Dow Jones PT GK Volatility with Breaks

We present the estimation results for the DJ index in Table 8 (similar results for the other four indices available upon request), where we choose to use the 3 breaks of the power transformed realized variance series: (1) 06/08/2003: pre-crisis break, (2) 30/10/2007: crisis break and (3) 20/12/2011: post-crisis break. The three dummies multiplied by the respective Heavy, Arch and Garman variables (to construct the slope dummies) are defined as follows: $D_{i,t} = 0$, if $t < T_i$ and $D_{i,t} = 1$, if $t \geqslant T_i$, $i = (1), (2), (3)$ the three break dates. The 3D-DAP specification with structural breaks consists of the following equations (superscripts in parentheses indicate the break date):

$$\begin{gathered}(1-\beta_r L)(\sigma_{rt}^2)^{\frac{\delta_r}{2}} = \omega_r + \\ (\gamma_{rr} + \gamma_{rr}^{(1)} D_{1,t-1} + \gamma_{rr}^{(2)} D_{2,t-1} + \gamma_{rr}^{(3)} D_{3,t-1}) s_{t-1} L(r_t^2)^{\frac{\delta_r}{2}} + \\ (\gamma_{rR} + \gamma_{rR}^{(1)} D_{1,t-1} + \gamma_{rR}^{(2)} D_{2,t-1} + \gamma_{rR}^{(3)} D_{3,t-1}) s_{t-1} L(RM_t)^{\frac{\delta_R}{2}} + \\ (\alpha_{rg} + \alpha_{rg}^{(1)} D_{1,t-1} + \alpha_{rg}^{(2)} D_{2,t-1} + \alpha_{rg}^{(3)} D_{3,t-1}) L(GK_t)^{\frac{\delta_g}{2}}\end{gathered} \tag{6}$$

$$
\begin{aligned}
(1-\beta_R L)(\sigma^2_{Rt})^{\frac{\delta_R}{2}} = \omega_R + [\alpha_{RR} + \alpha^{(1)}_{RR} D_{1,t-1} + \alpha^{(2)}_{RR} D_{2,t-1} + \alpha^{(3)}_{RR} D_{3,t-1} + \\
(\gamma_{RR} + \gamma^{(1)}_{RR} D_{1,t-1} + \gamma^{(2)}_{RR} D_{2,t-1} + \gamma^{(3)}_{RR} D_{3,t-1}) s_{t-1}] L(RM_t)^{\frac{\delta_R}{2}} + \\
(\gamma_{Rr} + \gamma^{(1)}_{Rr} D_{1,t-1} + \gamma^{(2)}_{Rr} D_{2,t-1} + \gamma^{(3)}_{Rr} D_{3,t-1}) s_{t-1} L(r^2_t)^{\frac{\delta_r}{2}} + \\
(\alpha_{Rg} + \alpha^{(1)}_{Rg} D_{1,t-1} + \alpha^{(2)}_{Rg} D_{2,t-1} + \alpha^{(3)}_{Rg} D_{3,t-1}) L(GK_t)^{\frac{\delta_g}{2}}
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
(1-\beta_g L)(\sigma^2_{gt})^{\frac{\delta_g}{2}} = \omega_g + \\
(\gamma_{gg} + \gamma^{(1)}_{gg} D_{1,t-1} + \gamma^{(2)}_{gg} D_{2,t-1} + \gamma^{(3)}_{gg} D_{3,t-1}) s_{t-1} L(GK_t)^{\frac{\delta_g}{2}} + \\
(\alpha_{gR} + \alpha^{(1)}_{gR} D_{1,t-1} + \alpha^{(2)}_{gR} D_{2,t-1} + \alpha^{(3)}_{gR} D_{3,t-1}) L(RM_t)^{\frac{\delta_R}{2}} + \\
(\gamma_{gr} + \gamma^{(1)}_{gr} D_{1,t-1} + \gamma^{(2)}_{gr} D_{2,t-1} + \gamma^{(3)}_{gr} D_{3,t-1}) s_{t-1} L(r^2_t)^{\frac{\delta_r}{2}},
\end{aligned} \tag{8}
$$

We firstly apply the slope dummies in the Heavy, Arch, and Garman parameters of the DAP-HEAVY-r equation (see Panel A). In the returns equation, we estimate three different specifications with breaks: the first (I) with the slope dummies on the cross Garman parameter, α_{rg}, the second (II) with the slope dummies on the own asymmetry (Arch) parameter, γ_{rr}, and the third (III) on the cross asymmetry (Heavy) parameter, γ_{rR}. All parameters increase with the crisis dummy and decrease with the pre- and post-crisis breaks. Regarding the realized measure equation (see Panel B), the Heavy impact, as captured by the Heavy parameter α_{RR}, and the own asymmetry γ_{RR}, the Arch asymmetric influence (captured by γ_{Rr}), and the Garman effect (α_{Rg}), all fall pre- and post-crisis and rise with the crisis break (specifications: I, II, III,IV). Finally, in the GK equation (Panel C), the own and the cross Arch asymmetries, γ_{gg} and γ_{gr}, and the Heavy impact, α_{gR}, increase during crisis and decrease during the pre- and post-crisis periods (specifications: I, II, III).

Table 8. The 3D-DAP-HEAVY model with structural breaks for Dow Jones

				Panel A: Stock Returns				
I	β_r	α_{rg}	$\alpha_{rg}^{(1)}$	$\alpha_{rg}^{(2)}$	$\alpha_{rg}^{(3)}$	γ_{rr}	γ_{rR}	
	0.80 (41.55)***	0.12 (4.71)***	−0.04 (−3.41)***	0.04 (3.15)***	−0.03 (−2.41)**	0.08 (4.75)***	0.11 (5.06)***	
II	β_r	α_{rg}	γ_{rr}	$\gamma_{rr}^{(1)}$	$\gamma_{rr}^{(2)}$	$\gamma_{rr}^{(3)}$	γ_{rR}	
	0.81 (44.92)***	0.10 (4.27)***	0.09 (4.37)***	−0.05 (−2.63)***	0.07 (3.33)***	−0.03 (−1.86)*	0.10 (4.85)***	
III	β_r	α_{rg}	γ_{rr}	γ_{rR}	$\gamma_{rR}^{(1)}$	$\gamma_{rR}^{(2)}$	$\gamma_{rR}^{(3)}$	
	0.81 (44.74)***	0.10 (4.22)***	0.08 (4.74)***	0.13 (5.08)***	−0.05 (−2.81)***	0.05 (2.95)***	−0.04 (−2.22)**	
				Panel B: Realized Measure				
I	β_R	α_{RR}	$\alpha_{RR}^{(1)}$	$\alpha_{RR}^{(2)}$	$\alpha_{RR}^{(3)}$	α_{Rg}	γ_{RR}	γ_{Rr}
	0.71 (39.24)***	0.10 (5.12)***	−0.02 (−3.22)***	0.02 (4.66)***	−0.03 (−6.08)***	0.12 (7.66)***	0.06 (5.53)***	0.08 (8.25)***
II	β_R	α_{RR}	α_{Rg}	$\alpha_{Rg}^{(1)}$	$\alpha_{Rg}^{(2)}$	$\alpha_{Rg}^{(3)}$	γ_{RR}	γ_{Rr}
	0.71 (38.87)***	0.08 (4.62)***	0.13 (8.47)***	−0.02 (−3.43)***	0.03 (5.07)***	−0.04 (−6.40)***	0.06 (5.55)***	0.08 (8.34)***
III	β_R	α_{RR}	α_{Rg}	γ_{RR}	$\gamma_{RR}^{(1)}$	$\gamma_{RR}^{(2)}$	$\gamma_{RR}^{(3)}$	γ_{Rr}
	0.71 (40.47)***	0.09 (4.70)***	0.13 (8.04)***	0.06 (5.11)***	−0.02 (−1.79)*	0.04 (3.70)***	−0.05 (−4.69)***	0.08 (8.15)***
IV	β_R	α_{RR}	α_{Rg}	γ_{RR}	γ_{Rr}	$\gamma_{Rr}^{(1)}$	$\gamma_{Rr}^{(2)}$	$\gamma_{Rr}^{(3)}$
	0.72 (39.97)***	0.08 (4.45)***	0.13 (8.13)***	0.05 (5.18)***	0.08 (8.22)***	−0.01 (−1.68)*	0.03 (2.95)***	−0.04 (−3.29)***
				Panel C: GK volatility				
I	β_g	γ_{gg}	$\gamma_{gg}^{(1)}$	$\gamma_{gg}^{(2)}$	$\gamma_{gg}^{(3)}$	α_{gR}	γ_{gr}	
	0.77 (35.21)***	0.11 (9.13)***	−0.04 (−4.13)***	0.02 (2.25)**	−0.03 (−3.83)***	0.09 (6.75)***	0.05 (6.83)***	
II	β_g	γ_{gg}	α_{gR}	$\alpha_{gR}^{(1)}$	$\alpha_{gR}^{(2)}$	$\alpha_{gR}^{(3)}$	γ_{gr}	
	0.76 (34.40)***	0.08 (8.38)***	0.11 (7.23)***	−0.03 (−5.19)***	0.02 (3.29)***	−0.02 (−5.54)***	0.06 (7.13)***	
III	β_g	γ_{gg}	α_{gR}	γ_{gr}	$\gamma_{gr}^{(1)}$	$\gamma_{gr}^{(2)}$	$\gamma_{gr}^{(3)}$	
	0.77 (34.94)***	0.08 (7.96)***	0.10 (6.77)***	0.08 (8.14)***	−0.03 (−3.64)***	0.01 (1.70)*	−0.02 (−2.25)**	
				Powers δ_i				
	δ_r	δ_R	δ_g					
	1.30	1.10	1.00					

Notes: See notes in Table 2.

All in all, we evidence consistently the same signs of the dummies coefficients across all specifications with Heavy, Arch, and Garman parameters. The dummy parameters corresponding to the 2003 and 2011 breaks are negative, whereas the ones for the 2007/08 crisis are positive.

8 Conclusions

Our study has extended the bivariate HEAVY system to the three-dimensional HYDAP specification. Our major contribution to volatility modeling research within this HEAVY framework is twofold: We, firstly, augment the benchmark model with a third variable, that is the range-based volatility, in order to achieve greater accuracy in volatility forecasting. Secondly, we enrich the trivariate formulation by taking into consideration leverage, power transformations, and long memory characteristics. For the realized measure, our empirical results favor the most general hyperbolic asymmetric power specification, where the lags of all three powered variables - squared negative returns, GK volatility, and realized variance - move the dynamics of the power transformed conditional variance of the latter. Similarly, modeling the returns with a fractionally integrated asymmetric power process, we found that not only the powered realized measure and negative signed GK volatility but the power transformed squared negative returns, as well, help to forecast the conditional variance of the latter.

The hyperbolic or fractionally integrated long memory of volatility, its asymmetric response to negative and positive shocks and its power transformations ensure the superiority of our contribution, which can be implemented on the areas of asset allocation and portfolio selection, as well as on several risk management practices. Further, we provided evidence on the forecasting superiority of our extensions over the benchmark HEAVY model through the rolling window out-of-sample forecasting across multiple short- and long-term horizons. Finally, the detection of structural breaks and the inclusion of break dummies in the asymmetric power formulation captures the time-varying pattern of the parameters, as the break corresponding to the financial crisis of 2007/08, in particular, consistently increases the values of the Heavy, Arch, and Garman parameters.

Our empirical findings on the nexus between low-frequency daily squared returns, range-based volatility, and high-frequency intra-daily realized measures, provide a volatility forecasting framework with important implications for policymakers and market practitioners, from investors, risk and portfolio managers up to financial chiefs, leaving ample room for future research on further model extensions. Thereupon, policymakers and market players should use our HEAVY framework to closely watch and forecast financial volatility patterns in the process of devising drastic policies, enforcing the financial system's regulations to preserve financial stability, deciding on asset allocation, hedging strategies, and investment projects. As part of future research, it would be interesting to extend our study to exchange rate market volatility and several other asset classes. A further interesting line of future research could be the extension of the multivariate HEAVY formulation of Noureldin et al. (2012) with leverage, power transformations and long memory, starting from the recent study of Dark (2018), who has applied a long memory multivariate GARCH model to the multivariate HEAVY, or Opschoor et al. (2018) within the

Generalized Autoregressive Score (GAS) process of Creal et al. (2013).

Data Availability Statement

The data that support the findings of this study are publicly available in the Oxford-Man Institute Realized Library at https://realized.oxford-man.ox.ac.uk/data/download.

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A APPENDIX: 3D-Benchmark Model Results

Table A.1. The 3D-Benchmark HEAVY model

	DJ	KOSPI	CAC	AORD	IPC
			Panel A: Stock Returns, HEAVY-r		
			$(1-\beta_r L)\sigma^2_{rt} = \omega_r + \alpha_{rR} L(RM_t) + \alpha_{rg} L(GK_t)$		
β_r	0.68 (17.59)***	0.67 (10.81)***	0.44 (7.65)***	0.77 (25.67)***	0.91 (61.24)***
α_{rR}	0.18 (3.36)***	0.40 (2.99)***	0.76 (6.66)***	0.28 (4.48)***	0.07 (6.78)***
α_{rg}	0.23 (4.41)***	0.23 (1.86)*	0.06 (3.52)***	0.13 (2.26)**	0.20 (7.21)***
Q_{12}	16.89 [0.15]	11.83 [0.46]	12.19 [0.43]	15.27 [0.23]	16.90 [0.15]
SBT	3.13 [0.00]	2.53 [0.01]	2.35 [0.02]	2.59 [0.01]	4.60 [0.00]
lnL	−6315.85	−7579.14	−7757.28	−5721.07	−7398.91
			Panel B: Realized Measure, HEAVY-R		
			$(1-\beta_R L)\sigma^2_{Rt} = \omega_R + \alpha_{RR} L(RM_t) + \alpha_{Rg} L(GK_t)$		
β_R	0.58 (12.42)***	0.55 (13.81)***	0.57 (16.89)***	0.73 (28.21)***	0.67 (11.04)***
α_{RR}	0.31 (4.44)***	0.34 (8.33)***	0.36 (9.78)***	0.19 (7.20)***	0.26 (3.49)***
α_{Rg}	0.14 (4.18)***	0.11 (3.95)***	0.06 (2.88)***	0.09 (3.82)***	0.06 (2.66)***
Q_{12}	12.85 [0.38]	15.44 [0.22]	9.46 [0.66]	16.89 [0.15]	9.53 [0.48]
SBT	3.45 [0.00]	5.26 [0.00]	2.39 [0.02]	2.67 [0.01]	3.12 [0.00]
lnL	−5922.35	−6135.93	−6818.17	−4357.03	−5816.53
			Panel C: GK volatility, HEAVY-g		
			$(1-\beta_g L)\sigma^2_{gt} = \omega_g + \alpha_{gR} L(RM_t)$		
β_g	0.58 (12.13)***	0.50 (7.36)***	0.57 (13.46)***	0.75 (31.27)***	0.76 (14.33)***
α_{gR}	0.33 (7.67)***	0.46 (7.04)***	0.38 (9.84)***	0.20 (9.86)***	0.24 (4.44)***
Q_{12}	9.65 [0.65]	12.72 [0.24]	9.33 [0.67]	12.39 [0.26]	9.62 [0.66]
SBT	4.42 [0.00]	3.01 [0.00]	2.85 [0.00]	3.22 [0.00]	8.70 [0.00]
lnL	−5402.41	−6068.15	−6630.57	−3997.18	−6290.51

Notes: See notes in Table 2.