Futures and Forwards, Problem Set, Solutions

QUESTION 1:

$S = 30$

$r = 0.05$

The forward price is given by:

$$F = Se^{2r}$$

$F = 34.508$

If $F = 35$, the investors will sell the forward, and they will buy the stock by borrowing an amount of money equal to its price.

At the delivery date, their position will be: $+F = 35$ (selling the forward), $-Se^{2r} = 34.508$ (the compounding value of the loan). Their overall position is positive.

If $F = 31$, the investors will buy the forward, and they will sell the stock and invest this amount of money.

At the delivery date, their position will be: $-F = 31$ (buying the forward), $+Se^{2r} = 34.508$ (the compounding value of the investment). Their overall position is positive.

QUESTION 2:

The present value of the future cash flows (the two coupon payments), I, is given by:

$C_1 = 40$

$r_1 = 0.09$

$r_2 = 0.1$

$$I = C_1 e^{-r_1} + C_1 e^{-r_2}$$

$I = 72.751$

$P = 900$

Therefore, the forward price is given by:

$$F = (P - I)e^{r_2}$$

: $F = 914.25$

If $F = 930$ then the investors will sell the forward, and will buy the bond by borrowing an amount of money equal to the price of the bond.

At present, their overall position will be 0: $P - P$.

After one year their position will be: $+F$ (selling the forward), $-Pe^{r_2}$ (repaying the amount that borrowed at present), $+Ie^{r_2}$ (the compounding value of the present value of the future cash flows).

Notice that $F - (P - I)e^{r_2} = 930 - 914.25 = 15.75 > 0$.

If $F = 905$ then the investors will buy the forward, and will sell the bond and invest this amount of money.

After one year their position will be: $-F$ (buying the forward), $+Pe^{r_2}$ (the compounding value of the bond), $-Ie^{r_2}$ (the compounding value of the present value of the future cash flows).

Notice that $-F + (P - I)e^{r_2} = -905 + 914.25 = 9.25 > 0$.

QUESTION 3:

$S = 25$
$r = 0.1$
$S_1 = 24$

$$F = Se^{\frac{1}{2}r}$$

: $F = 26.282$

The value of the contract is:

$$f = (S_1 - F)e^{-\frac{1}{2}r}$$

: $f = -2.1707$

QUESTION 4:

$G = 450$

$r = 0.07$

$U = 2e^{-r}$, $U = 1.8648$ (the present value of the storage cost)

$$F = (G + U)e^{r}$$

$F = 484.63$

QUESTION 6:
$\sigma_1 = 0.032$
$\sigma_2 = 0.040$
$\rho = 0.8$
The hedge coefficient is given by:

$$\beta = \frac{\rho\sigma_1}{\sigma_2}$$

: $\beta = 0.64$

$Q = 1000000$
Therefore, the position taken in the futures contracts is:

$$H = Q\beta$$

: $H = 6.4 \times 10^5 = 640000$
Thus, the optimal number of futures contracts is:

$$\frac{640000}{42000}$$

: 15.238