Problem Set: B&S, Solutions
Black and Scholes formula for the call:

$$
\begin{aligned}
d_1 &= \frac{\ln(\frac{S_t}{X}) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}, \\
d_2 &= \frac{\ln(\frac{S_t}{X}) + (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}, \\
N(d_1) &= \\
N(d_2) &= \\
C &= S_t N(d_1) - X e^{-r(T-t)} N(d_2).
\end{aligned}
$$

Numerical formulae:

Question 1:
$S = 52$
$X = 50$
$r = 0.12$
$\sigma = 0.09$ $\rightarrow \sigma^2 = (0.3)^2 = 0.09$
$T = 0.25$

$\rightarrow \underbrace{\sigma^2 = 0.09}$

$$
\begin{aligned}
d_1 &= \frac{\ln(\frac{S}{X}) + (r + \frac{1}{2}\sigma)(T)}{\sqrt{\sigma T}}, \\
d_2 &= \frac{\ln(\frac{S}{X}) + (r - \frac{1}{2}\sigma)(T)}{\sqrt{\sigma T}}, \text{ or } d_2 = 0.53647 - \sqrt{\sigma T} \\
N_1 &= 0.7 \\
N_2 &= 0.652 \\
C &= SN_1 - Xe^{-r(T)}N_2.
\end{aligned}
$$

$N(d_1)$

$N(d_2)$

$d_2 = 0.386\,47 : \ d_1 = 0.536\,47$:

$C = 4.763\,5$

Question 2:

C=2.5
$S = 15$
$X = 13$
$r = 0.05$
$\sigma = 0.09 \quad \sigma = 0.3 \quad \sigma^2 = 0.09$
$T = 0.25$

$T-t$

σ^2

$$d_1 = \frac{\ln(\frac{S}{X}) + (r + \frac{1}{2}\sigma)(T)}{\sqrt{\sigma T}}, \quad T-t$$

$N(d_1)$

$$d_2 = \frac{\ln(\frac{S}{X}) + (r - \frac{1}{2}\sigma)(T)}{\sqrt{\sigma T}},$$

$$N_1 = 0.8413$$

$N(d_2)$

$$N_2 = 0.8315$$

$$C = SN_1 - Xe^{-r(T)}N_2.$$

$\sigma = 0.3$

$C = 1.9443 : d_2 = 0.96234 : d_1 = 1.1123$

but we want $C = 2.5$

We should increase σ

$\sigma^2 \quad \sigma \quad \sigma = 0.16 \quad \sigma = 0.4 \quad \sigma^2 = (0.4)^2 = 0.16$

C=2.5
$S = 15$
$X = 13$
$r = 0.05$
$\sigma^2 \quad \sigma \quad \sigma = 0.16 \quad \sigma = 0.4 \quad \sigma^2 = 0.16$
$T = 0.25$

$$d_1 = \frac{\ln(\frac{S}{X}) + (r + \frac{1}{2}\sigma)(T)}{\sqrt{\sigma T}},$$

$$d_2 = \frac{\ln(\frac{S}{X}) + (r - \frac{1}{2}\sigma)(T)}{\sqrt{\sigma T}},$$

$$N_1 = 0.8106$$

$$N_2 = 0.7517$$

$$C = SN_1 - Xe^{-r(T)}N_2.$$

$C = 2.5083 : d_2 = 0.678 : d_1 = 0.878$

δ^2

We should decrease σ
$\sigma = 0.1225 \quad \sigma = 0.35 \quad \sigma^2 = (0.35)^2 = 0.1225$

C=2.5
$S = 15$
$X = 13$
$r = 0.05$
$\sigma = 0.1225 \quad \sigma = 0.35 \quad \sigma^2 = 0.122\,5$
$T = 0.25$

$$\begin{aligned} d_1 &= \frac{\ln(\frac{S}{X}) + (r + \frac{1}{2}\sigma)(T)}{\sqrt{\sigma T}}, \\ d_2 &= \frac{\ln(\frac{S}{X}) + (r - \frac{1}{2}\sigma)(T)}{\sqrt{\sigma T}}, \\ N_1 &= 0.8365 \\ N_2 &= 0.7881 \\ C &= SN_1 - Xe^{-r(T)}N_2. \end{aligned}$$

$\sigma = 0.35$

$: \ C = 2.429\,5 \ : \ d_2 = 0.801\,65 \ : \ d_1 = 0.976\,65$
So the implied volatility is between 0.35 and 0.40.

Question 3:

$S = 30$
$X = 29$
$r = 0.05$
$\sigma = 0.0625$ $\sigma = 0.25$ $\sigma^2 = (0.25)^2 = 0.062\,5$
σ^2 $T = 0.3$

σ^2

$$\begin{aligned} d_1 &= \frac{\ln(\frac{S}{X}) + (r + \frac{1}{2}\sigma)(T)}{\sqrt{\sigma T}}, \\ d_2 &= \frac{\ln(\frac{S}{X}) + (r - \frac{1}{2}\sigma)(T)}{\sqrt{\sigma T}}, \\ N_1 &= 0.6628 \\ N_2 &= 0.6141 \\ C &= SN_1 - Xe^{-r(T)}N_2. \end{aligned}$$

σ^2

$: C = 2.340\,2 : d_2 = 0.288\,66 : d_1 = 0.425\,59$
The price for the American call is the same.
The price of the put, using the put-call parity is:

$$P = C + Xe^{-r(T)} - S$$

$: P = 0.908\,45$

Question 4:

$S = 40$
$X = 40$
$r = 0.05$
$\sigma = 0.04 \quad \sigma = 0.20 \quad \sigma^2 = (0.20)^2 = 0.04$
$T = 0.0833$

$$
\begin{aligned}
d_1 &= \frac{\ln(\frac{S}{X}) + (r + \frac{1}{2}\sigma)(T)}{\sqrt{\sigma T}}, \\
d_2 &= \frac{\ln(\frac{S}{X}) + (r - \frac{1}{2}\sigma)(T)}{\sqrt{\sigma T}}, \\
N_1 &= 0.5398 \\
N_2 &= 0.5 \\
C &= SN_1 - Xe^{-r(T)}N_2.
\end{aligned}
$$

$: C = 1.6751 \;\; d_2 = 4.3293 \times 10^{-2} : d_1 = 0.10102$

Using the put-call parity, the price of the put is:

$$P = C + Xe^{-r(T)} - S$$

$: P = 1.5088$
The price of the call exceeds the price of the put because:

$Xe^{-r(T)} = 39.834$ is less than the price of the stock $S = 40$.
Alternatively, the price of the put can be calculated as:

$$
\begin{aligned}
d_1 &= \frac{\ln(\frac{S}{X}) + (r + \frac{1}{2}\sigma)(T)}{\sigma\sqrt{T}}, \\
d_2 &= \frac{\ln(\frac{S}{X}) + (r - \frac{1}{2}\sigma)(T)}{\sigma\sqrt{T}}, \\
N_1 &= 0.5398 \\
N_2 &= 0.5 \\
C &= SN_1 - Xe^{-r(T)}N_2, \\
P &= -S(1 - N_1) + Xe^{-r(T)}(1 - N_2)
\end{aligned}
$$

$: P = 1.5089$

Question 5:

The relation between annualized and continuously compounding is given by:

$$\begin{aligned} (1+r_a) &= e^{r_c} \Longrightarrow \\ \ln(1+r_a) &= r_c, \end{aligned}$$

$r = 0.06$

$\ln(1+r) = 5.8269 \times 10^{-2} = 0.058269$