J. Int. Financ. Markets Inst. Money 91 (2024) 101943

Contents lists available at ScienceDirect

Journal of International Financial Markets, Institutions & Money

journal homepage: www.elsevier.com/locate/intfin

The efficiency of the Estr overnight index swap market

Marco Realdon

Economics and Finance Department, Brunel University London, Kingstone Lane, Uxbridge, UB8 3PH, UK

ARTICLE INFO

JEL classification:
G12
G13

Keywords:
Estr overnight index swaps
Affine pricing models
Pricing errors
Delta-hedging
Market-neutral arbitrage portfolios
Transaction costs

ABSTRACT

This paper studies the profitability of market-neutral delta-hedged strategies trading the mispricing of Euro Short Term Rate Overnight Index Swaps (Estr OIS) signalled by standard affine term structure models. Calibrating these models produces pricing errors that signal mispricing and the deltas to hedge market risk. The paper presents simple-to-compute portfolio weights that maximise the OIS arbitrage portfolio information ratio subject to market-neutral delta-hedge constraints and subject to bid–ask spreads. The empirical evidence shows that only investors who can "split" the bid–ask spread can profitably exploit the pricings errors signalled by these models. Investors who can only ever trade at the bid or at the ask cannot profit. Pricing errors are strongly positively auto-correlated, which hampers the profitability of trades that expect the correction of such errors. These results imply that the Estr OIS market is quite efficient and are robust to a number of models and strategies. Four and five factor models are more profitable than three factor ones. Assuming that some OIS rates are observed without error reduces the profitability of models and strategies.

1. Introduction

This paper explores the efficiency of a key swap market in the Euro area, namely the overnight index swap (OIS) market, which now references the Euro Short Term Rate (Estr), an overnight rate determined by the European Central Bank (ECB) since October 2019 and successor of the Euro Overnight Index Average (Eonia). Klingler and Syrstad (2021) provide an early empirical analysis of Estr. In 2022 the Estr OIS market by and large replaced the Eonia OIS market, itself a large market with notional outstanding of about 8.7 trillion Euros in 2020, as explained in Cera et al. (2020). Since October 2019 Eonia has been calculated as Estr plus a spread of 8.5 basis points. This calculation of Eonia is used in OIS that continue to reference Eonia after the cessation of Eonia in January 2022. In 2022 the Estr OIS market was already larger than the well established market for swaps that reference Euribor, as reported in Huang and Todorov (2022). Despite its size and importance, the young Estr OIS market is quite unexplored in the literature. Studies of the Estr OIS market efficiency seem almost absent so far.

This paper investigates the efficiency of the Estr OIS market since its inception in October 2019 through the lens of standard affine pricing models. The evidence shows that affine OIS pricing models can closely match the term structure of OIS prices each day. Pricing errors, i.e. the differences between model OIS rates and market OIS rates, tend to be two to three basis points. Evidence that these pricing errors signal mispricing that can profitably and consistently be exploited would be against the efficiency of the young Estr OIS market. The evidence in this paper based on tests of trading strategies shows that the Estr OIS market is quite efficient.

The arbitrage trading strategies we test are "long-short" Estr OIS of different maturities and are market-neutral, because they delta-hedge any exposure of the arbitrage portfolio to the latent factors driving Estr and Estr OIS rates. The arbitrage portfolio is rebalanced daily so as to maximise an information ratio (IR) for a one-day holding period. The IR is the expected one day portfolio profit divided by the standard deviation of such profit. Since the portfolio is market-neutral, portfolio profit is mainly driven by

E-mail address: marco.realdon@brunel.ac.uk.

https://doi.org/10.1016/j.intfin.2024.101943
Received 5 July 2023; Accepted 11 January 2024
Available online 26 January 2024

changes in the pricing errors of OIS of different maturities. A contribution of this paper is in computing portfolio weights that maximise the said information ratio subject to transaction costs and delta-hedge constraints.

The evidence shows that these trading strategies can be very profitable to investors who can always trade Estr OIS at the mid-price. This is so even as investors use nothing more than standard Gaussian affine models to price OIS. However none of this profitability is available to investors who can only ever trade at the bid and at the ask quotes. Some of the said profitability is available to investors who can "split" the bid–ask spread, for example because they have enough bargaining power toward market makers or because the bid and ask quotes are only indicative. For some affine models, strategies profitability can be attractive when the effective spread, i.e. the absolute value of the difference between mid price and trade price, is on average one eight or even one fourth of the bid–ask spread. However in these cases the profitability of strategies relies on very aggressive portfolio positions on few rare days, especially after OIS rates began to rise in 2021.

The trading strategies also shed light on the relative profitability of different specifications of affine pricing models. One question is whether increasing the number of factors in the OIS pricing model can improve profitability. With more factors, the pricing errors can be smaller, but remain strongly auto-correlated, which hampers the profitability of trading strategies. Then more factors entail more constraints to keep the trading portfolio delta-hedged and thus market-neutral. Portfolio weights that need to satisfy more delta-hedge constraints have less freedom to maximise the profitability of arbitrage strategies. The evidence shows that four and five factor models can be similarly profitable, and more profitable than three factor models. This contrasts with the statistical view, for example in Duffee (2013), that three factors suffice to model Government bond yields, because these yields are mainly driven by the first three principal components.

Another question concerns the assumption, common in the term structure literature, that some interest rates or combinations thereof are observed with no errors, as for example in Joslin et al. (2011). The evidence shows that models that assume that all OIS rates are observed with errors are more profitable and overall fit OIS rates more closely, than do models that assume that some of the OIS rates are observed without errors.

This paper is closest in spirit to a couple of papers that test the profitability of arbitrage portfolios made up of swaps. The first paper is Bali et al. (2009), who investigate the US Libor based interest rate swap market. The second paper is Jarrow et al. (2019), who investigate corporate credit default swaps (CDS). These papers find that their arbitrage portfolios can be profitable.

Bali et al. (2009) test market-neutral delta-hedged arbitrage portfolios of interest rate swaps (IRS) that exploit the mispricing of IRS signalled by Gaussian affine term structure models. Also the present paper employs delta-hedged arbitrage portfolios to exploit arbitrage opportunities signalled by Gaussian affine term structure models, but uses different arbitrage portfolio strategies, different Gaussian affine models, and uses Estr OIS rather than US Dollar IRS. Bali et al. (2009) specify affine models with up to three factors and estimate them with a type of Kalman Filter. Their arbitrage portfolios consist of up to four IRS. Instead the present paper specifies affine models with up to five factors, as more than three factors seem needed for OIS arbitrage. Some of these models use the observable Estr rate, some follow Heidari and Wu (2010) in accounting for changes in ECB policy rates, and all of these models are only specified under the risk-neutral measure and calibrated through non-linear least squares. The arbitrage portfolios comprise OIS of all yearly maturities from one year to ten years, and are opened and closed so as to maximise an information ratio for a one day holding period, which is a contribution of this paper. The information ratio is expected portfolio profit divided by the standard deviation of portfolio profit. Instead Bali et al. (2009) open portfolios of four IRS every week and keep them for four weeks. Their portfolio positions are long or short depending on whether the portfolio is overpriced or underpriced, and the amount invested in each swap portfolio is weighted. The weight is linear in the expected change of the portfolio pricing error and is divided by the conditional variance of the portfolio pricing error. They admit this "simple decision rule is far from being optimal" (Bali et al. (2009), page 524). Their sample is made of Libor rates of six different maturities and swap rates of nine different maturities from two years to thirty years. They find that their IRS arbitrage portfolios can be profitable. Instead this paper finds that Estr OIS arbitrage portfolios are not profitable to investors who can only ever trade at the bid and ask quotes, but may be profitable to investors who can "split" the bid–ask spreads of OIS. A possible explanation is that the Estr OIS market from 2019 to 2023 studied in this paper may be more efficient than the US Dollar IRS market from 1994 to 2007 studied in Bali et al. (2009). This paper differs from Bali et al. (2009) also in testing the profitability of specific features of affine models, such as the number of factors, the frequent assumption of perfectly observed interest rates, or the explicit modelling of jumps in Estr when ECB policy rates change.

Jarrow et al. (2019) test the profitability of market-neutral delta-hedged and gamma-hedged arbitrage portfolios involving US corporate CDS. Again the portfolios exploit the mispricing of CDS contracts as signalled by affine pricing models. They find that a two factor model is more profitable than a one factor model. Also Doshi et al. (2013) show the profitability and hedging effectiveness of corporate CDS pricing models with observable covariates, which they use to detect mispricing that is exploited through long-short CDS portfolios. More recently Rebonato and Ronzani (2021), among others, evaluate the profitability of long-short arbitrage trading strategies involving IRS of different maturities, but their strategies are not based on an arbitrage-free IRS pricing model.

Few papers have studied OIS markets. A notable study is Lloyd (2020), who shows how US interest rate expectations can be better estimated by augmenting US Treasury yields with OIS rates. Lloyd (2021) shows that OIS rates in US, UK, Euro area and Japan are reliable measures of rate expectations up to 2 years in the future. His Euro area sample uses Eonia OIS, not Estr OIS. Cousin et al. (2016) apply kriging, a machine learning technique, to interpolate discount factors extracted from Eonia OIS. Sundaresan et al. (2016) propose a term structure model for the US OIS market before the switch from Libor to Secured Overnight Financing Rate (Sofr).

From a methodology point of view this paper follows Ang and Longstaff (2013), in that pricing models are specified only under the pricing measure, not under the physical measure, and estimated by minimising squared pricing errors through non-linear least squares. This paper also follows Heidari and Wu (2010) in the way it models the predictable jumps in Estr after ECB policy rates change.

The paper is organised as follows. The next section presents the OIS pricing models. The following section presents the arbitrage strategies. Then the empirical results are illustrated, and the conclusions follow.

2. The OIS pricing models

This section presents the OIS pricing models tested in the empirical analysis. The models are specified only under the risk-neutral pricing measure $\mathbb{Q}$ and can signal market pricing errors, i.e. market mispricing of OIS contracts. Omitting to specify models under physical measure avoids issues due to explicitly modelling risk premia, or issues due to non-time-homogeneous processes underlying OIS data, structural breaks, regime changes, non-stationarity.

We split time into "trading days" of length Δ, with $\Delta = 1/260$ as time is measured in years and as there are about 260 trading days per year in our sample. Let $Z_{t,m}$ be the price of a discount bond on day t with maturity on day $t+m$ and with unit face value. Then the m-day discount bond yield on day t is $-\frac{\ln Z_{t,m}}{m\Delta}$. $r_t = -\frac{\ln Z_{t,1}}{\Delta}$ is the short interest rate on day t. Following Filipović and Trolle (2013), we use the approximation

$$1 + \Delta L_t \approx e^{r_t \Delta}$$

where L_t is the Estr index on day t as published by the ECB. Thanks to this approximation, hereafter we refer to r_t as Estr for day t. Let $Z_{t,m}$ be the price of a hypothetical discount bond whose price is computed using Estr as discount rate. Absent arbitrage

$$Z_{t,m} = E_t^{\mathbb{Q}}\left[e^{-\sum_{\tau=t}^{t+m-1} r_\tau \Delta}\right].$$

$E_t^{\mathbb{Q}}$ [..] denotes the conditional expectation on day t under the $\mathbb{Q}$ measure. Let $OIS_{M,t}$ denote the model-predicted OIS rate on day t for the M-year OIS contract. Estr OIS payments are exchanged at the end of every year during the contract life, if the OIS maturity is no less than one year. Our sample comprises OIS of ten yearly maturities from one year to ten years, so that $M = 1,2,3,4,5,6,7,8,9,10$. Then following Filipović and Trolle (2013) we use the approximation

$$\overline{L}_j = \frac{\prod_{\tau=t+\Delta^{-1}(j-1)}^{t+\Delta^{-1}j-1}\left(1+\Delta L_\tau\right)-1}{\delta} \approx \frac{\exp\left(\sum_{\tau=t+\Delta^{-1}(j-1)}^{t+\Delta^{-1}j-1} r_\tau \Delta\right)-1}{\delta}$$

where $\overline{L}_j$ is the compounded Estr rate for the year $\left[t+\Delta^{-1}(j-1), t+\Delta^{-1}j\right]$. Note that $\delta = \Delta\left(t+\Delta^{-1}j-\left(t+\Delta^{-1}(j-1)\right)\right) = 1$. Then again following Filipović and Trolle (2013) it can be shown that, absent arbitrage, OIS rates can be computed as

$$OIS_{M,t} = \frac{\delta \cdot \sum_{j=1}^{M} E_t^{\mathbb{Q}}\left[e^{-\sum_{\tau=t}^{t+\Delta^{-1}j-1} r_\tau \Delta} \cdot \overline{L}_j\right]}{\delta \cdot \sum_{j=1}^{M} E_t^{\mathbb{Q}}\left[e^{-\sum_{\tau=t}^{t+\Delta^{-1}j-1} r_\tau \Delta}\right]} = \frac{1 - Z_{t,\Delta^{-1}\cdot M}}{\delta \cdot \sum_{j=1}^{M} Z_{t,\Delta^{-1}\cdot j}}.$$

We assume an Gaussian affine term structure model such that

$$r_t = \boldsymbol{\rho}'\mathbf{x}_t = x_{n,t}$$
$$\boldsymbol{\rho} = \mathbf{e}_{n,n}$$

where $\mathbf{x}_t = \left(x_{1,t}, \ldots, x_{n,t}\right)'$ is a $n \times 1$ vector of latent factors on day t. $\mathbf{e}_{i,n}$ is the ith column of the $n \times n$ identity matrix $\mathbf{I}_n$. Thus $\mathbf{e}_{n,n}$ is the nth column of $\mathbf{I}_n$. We assume that the conditional density of $\mathbf{x}_{t+1}$ under $\mathbb{Q}$ is Gaussian, that $n = 3$ or $n = 4$ or $n = 5$ and that

$$\begin{aligned} &E_t^{\mathbb{Q}}\left[\mathbf{x}_{t+1}\right] - \mathbf{x}_t = \left(\boldsymbol{\mu} - \boldsymbol{\kappa}\mathbf{x}_t\right) \cdot \Delta \\ &var_t^{\mathbb{Q}}\left[\mathbf{x}_{t+1}\right] = \left(diag\left(\boldsymbol{\sigma}\right)\right)^2 \cdot \Delta \\ &\boldsymbol{\mu} = \mathbf{e}_{1,n} \cdot \mu_1, \quad \mu_1 \geq 0 \\ &\boldsymbol{\kappa} = \sum_{i=1}^{n} \mathbf{e}_{i,n} \cdot \mathbf{e}'_{i,n} \cdot \kappa_i - \sum_{i=2}^{n} \mathbf{e}_{i,n} \cdot \mathbf{e}'_{i-1,n}. \end{aligned} \tag{1}$$

$\boldsymbol{\mu}$,$\boldsymbol{\sigma}$ are $n \times 1$ vectors and $\boldsymbol{\kappa}$ is a $n \times n$ matrix of parameters. $diag(\boldsymbol{\sigma})$ denotes the diagonal matrix whose main diagonal is $\boldsymbol{\sigma} = \left(\sigma_1, \ldots, \sigma_n\right)'$. $var_t^{\mathbb{Q}}\left[\mathbf{x}_{t+1}\right]$ denotes the day t conditional variance of $\mathbf{x}_{t+1}$ under $\mathbb{Q}$. Conditions (1) are chosen because of their good empirical performance and parsimony. The conditions $\kappa_i > 0$ for $i = 1, \ldots, n$ are necessary for r_t to be stationary. It is well known that under the above assumptions $Z_{t,m} = \exp\left(A_m + \mathbf{B}'_m \mathbf{x}_t\right)$, where

$$\begin{aligned} A_m &= A_{m-1} + \mathbf{B}'_{m-1}\boldsymbol{\mu}\Delta + \frac{1}{2}\Delta\mathbf{B}'_{m-1} \cdot \left(diag\left(\boldsymbol{\sigma}\right)\right)^2 \cdot \mathbf{B}_{m-1}, \quad A_0 = 0 \\ \mathbf{B}'_m &= -\boldsymbol{\rho}'\Delta + \mathbf{B}'_{m-1}\left(\mathbf{I}_n - \boldsymbol{\kappa}\Delta\right), \quad \mathbf{B}_0 = \mathbf{0}_{n\times 1}. \end{aligned}$$

$\mathbf{0}_{n\times 1}$ is an $n \times 1$ vector of elements all equal to 0.

2.1. Heidari and Wu (2010) model variant

We also estimate a variant of this model that follows Heidari and Wu (2010). In such variant we set $\Delta = 1/256$, as the number of Target2 business days, the days for which Estr is computed, is around 256 in one year. Setting $\Delta = 1/256$, rather than $\Delta = 1/260$ as above, has a negligible effect on the analysis. Then we assume 32 business days between one ECB Governing Council rate setting meeting and the next, since there are 8 such meetings per year and $256/8 = 32$. Then we set

$$\begin{aligned} \boldsymbol{\kappa} &= \sum_{i=1}^{n} \mathbf{e}_{i,n} \cdot \mathbf{e}'_{i,n} \cdot \ \kappa_i - \sum_{i=2}^{n-1} \mathbf{e}_{i,n} \cdot \mathbf{e}'_{i-1,n} - 1_{m=32\cdot q} \cdot \mathbf{e}_{n,n} \cdot \mathbf{e}'_{n-1,n} \\ \kappa_n &= 1_{m=32\cdot q} \cdot \overline{\kappa}_n \\ \sigma_n &= 1_{m=32\cdot q} \cdot \overline{\sigma}_n \end{aligned} \tag{2}$$

where $q = 1, 2, 3, \ldots$ $1_{m=32\cdot q}$ is the indicator function of the event $m = 32 \cdot q$. $\overline{\kappa}_n$ and $\overline{\sigma}_n$ are two parameters. The meaning of this model variant is similar to that of the Gaussian affine term structure model in Heidari and Wu (2010). r hardly changes from day to day, but jumps up or down on the days ECB policy rates change. Following Heidari and Wu (2010) the size of such jumps is random and distributed according to a Gaussian density, so that

$$\begin{aligned} E^{\mathbb{Q}}_{t+m-1}\left[r_{t+m}\right] &= r_{t+m-1} + 1_{m=32\cdot q} \cdot \left(x_{n-1,t+m-1} - \overline{\kappa}_n r_{t+m-1}\right) \cdot \Delta \\ var^{\mathbb{Q}}_{t+m-1}\left[r_{t+m}\right] &= \left(1_{m=32\cdot q} \cdot \overline{\sigma}_n\right)^2 \cdot \Delta. \end{aligned}$$

While Heidari and Wu (2010) use a similar approach to model jumps in the Fed Funds Target rate after each Federal Open Market Committee (FOMC) rate setting meeting, this paper models jumps in Estr after each ECB Governing Council rate setting meeting.

Other things equal, conditions (1) assume that Estr can change each day, while conditions (2) assume that Estr can jump only every 32 days. Note that the more persistent over time factor x_{n-1} is, the more auto-correlated the successive jumps of r tend to be, a fact that reflects "gradualism" of monetary policy.

Conditions (1) can approximate conditions (2) for the purpose of pricing OIS with maturities from 1 to 10 years, as in our sample. For these maturities computing OIS rates under conditions (1) and conditions (2) gives similar results. Note again that the model is specified only under the risk-neutral pricing measure $\mathbb{Q}$, and not under the physical measure.

In the empirical tests we impose conditions (2) in models that are calibrated to both Estr and OIS rates. However, since the trading strategies we test only involve OIS positions, and not overnight lending or borrowing at the Estr rate, we also calibrate models using only OIS rates and not Estr, in which case we impose conditions (1). Calibrating models only to OIS rates tends to increase models profitability, since the models are more free to fit OIS rates without having to fit also the Estr rate.

Hereafter we use the suffix HW to denote models in which the conditions of Eqs. (2) are imposed. For example A5-HW denotes such a model when the factors are 5, i.e. when $n = 5$.

2.2. The pricing errors

Below we test arbitrage strategies that use the pricing errors of the above OIS pricing models as indicators of mispricing. These tests are close in spirit to those in Bali et al. (2009), but their tests concern US dollar denominated interest rate swaps, their models and estimations are different, and their tradings strategies are different. We define

$$\mathbf{y}^o_t = \mathbf{y}_t + \boldsymbol{\varepsilon}_t$$

where:

– $\mathbf{y}^o_t = \left(.., y^o_{M,t}, ..\right)'$ for $M = 1, \ldots, N$ is the vector of observed OIS rates on day t for all yearly maturities; $N = 10$ since the longest OIS maturity in our sample is ten years;

– $\mathbf{y}_t = \left(.., y_{M,t}, ..\right)'$ is the vector of model-computed OIS rates on day t, so that $y_{M,t} = OIS_{M,t}$ for $M = 1, \ldots, N$;

– $\boldsymbol{\varepsilon}_t$ is the $N \times 1$ vector of pricing errors on day t.

We assume that pricing errors follow the autoregressive process

$$\begin{aligned} \boldsymbol{\varepsilon}_{t+1} &= \boldsymbol{\Phi} \cdot \boldsymbol{\varepsilon}_t + \boldsymbol{\Sigma}_\varepsilon \cdot \boldsymbol{\epsilon}_{t+1} \\ E_t\left[\boldsymbol{\epsilon}_{t+1}\right] &= \mathbf{0}_{N\times 1} \\ var_t\left[\boldsymbol{\epsilon}_{t+1}\right] &= \mathbf{I}_N. \end{aligned}$$

$\mathbf{0}_{N\times 1}$ is an $N \times 1$ vector of zeros. $\boldsymbol{\epsilon}_{t+1}$ are random shocks on day $t+1$. $\boldsymbol{\Phi}$ and $\boldsymbol{\Sigma}_\varepsilon$ are matrices of parameters. $\mathbf{I}_N$ is the $N \times N$ identity matrix. Therefore

$$\mathbf{m}_{t,h} = E_t\left[\boldsymbol{\varepsilon}_{t+h} - \boldsymbol{\varepsilon}_t\right] = \left(\boldsymbol{\Phi}^h - \mathbf{I}_N\right) \cdot \boldsymbol{\varepsilon}_t$$

$$\begin{aligned} \boldsymbol{\Sigma}_{t,h} &= var_t\left[\boldsymbol{\varepsilon}_{t+h}\right] = \boldsymbol{\Phi} \cdot var_t\left[\boldsymbol{\varepsilon}_{t+h-1}\right] \cdot \boldsymbol{\Phi}' + \boldsymbol{\Sigma}_\varepsilon \cdot var_{t+h-1}\left[\boldsymbol{\epsilon}_{t+h}\right] \cdot \boldsymbol{\Sigma}'_\varepsilon \\ &= \sum_{i=1}^{h} \boldsymbol{\Phi}^{i-1} \cdot \boldsymbol{\Sigma}_\varepsilon \boldsymbol{\Sigma}'_\varepsilon \cdot \left(\boldsymbol{\Phi}'\right)^{i-1}. \end{aligned}$$

Note that $\boldsymbol{\Sigma}_{t,1} = var_t\left[\boldsymbol{\varepsilon}_{t+1}\right] = \boldsymbol{\Sigma}_\varepsilon \boldsymbol{\Sigma}'_\varepsilon$. We assume $\boldsymbol{\Phi} = diag\left(\boldsymbol{\phi}\right)$, where $\boldsymbol{\phi}$ is a conformable vector.

2.3. Measuring trading profit

Let $\pi_{t,h}$ denote the portfolio profit during the holding period $[t, t+h]$ from day t to day $t+h$. We set the holding period equal to one day, so that $h = 1$. We can use two approximate measures of $\pi_{t,1}$. The first measure is valuation-model-independent and is

$$\pi_{t,1} = \mathbf{w}_t' \cdot diag(\mathbf{M}) \cdot \left(\mathbf{y}_{t+1}^o - \mathbf{y}_t^o\right) - \mathbf{c}_t' \cdot \left|\mathbf{w}_t - \mathbf{w}_{t-1}\right|$$
$$\mathbf{M} = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10)'$$
$$\mathbf{c}_t = \alpha \cdot \mathbf{s}_t$$

where:

– $\mathbf{w}_t = \left(w_{1,t}, \dots, w_{10,t}\right)'$ is the 10×1 vector of portfolio weights at the beginning of day t;
– $\mathbf{M}$ is the set of OIS contract maturities from one year to ten years;
– $y_{M,t}^o \cdot M$ is the total nominal amount of fixed payments during the OIS contract's life $\left[t, t + M \cdot \Delta^{-1}\right]$, with $M = 1, 2, 3, \dots, 10$;
– we use $\left(y_{M,t+1}^o - y_{M,t}^o\right) \cdot M \cdot w_{M,t}$ to approximate $\left(y_{M,t+1}^o \cdot (M - \Delta) - y_{M,t}^o \cdot M\right) \cdot w_{M,t}$;
– $\mathbf{s}_t = \left(s_{1,t}, \dots, s_{10,t}\right)'$; $\mathbf{s}_t$ is the vector of bid–ask spreads for all OIS maturities at the beginning of day t; bid–ask spreads usually hover between 1 and 5 basis points in our sample; bid–ask spreads vary not only across OIS maturities, but also over time with market liquidity conditions;
– $\mathbf{c}_t = \left(c_{1,t}, \dots, c_{10,t}\right)'$ is a vector of "effective spreads" for all OIS maturities at the beginning of day t; the effective spread is the absolute value of the difference between mid price and trade price; we set $\mathbf{c}_t$ to be the fraction α of the bid–ask spreads $\mathbf{s}_t$; we experiment with $\alpha = 0, 1/2, 1/4, 1/8$; when $\alpha = 0$ the investor always trades at the mid price, as if the market was frictionless; when $\alpha = 1/2$ the investor always buys at the ask price and sells at the bid price; when $\alpha = 1/4, 1/8$ the investor can "split" the bid–ask spread;
– $\mathbf{c}_t' \cdot \left|\mathbf{w}_t - \mathbf{w}_{t-1}\right|$ with $\left|\mathbf{w}_t - \mathbf{w}_{t-1}\right| = \left(\left|w_{1,t} - w_{1,t-1}\right|, \dots, \left|w_{N,t} - w_{N,t-1}\right|\right)'$ are the transaction costs to rebalance the portfolio position at the beginning of day t; the portfolio is rebalanced at the beginning of each day.

In the empirical tests we compute how strategy profitability varies with transaction costs. We do this for a number of reasons. OIS bid and ask quotes may be just indicative. Actual transaction prices can be the result of some bargaining. Therefore the effective spread can differ from half of the bid–ask spread according to the identity and bargaining power of the counterparties. The empirical tests will show that transaction costs critically affect the profitability of trading strategies.

The second approximate measure of profit is $\overline{\pi}_{t,1}$ and is based on marking the arbitrage portfolio to market. $\overline{\pi}_{t,1}$ is valuation-model-dependent. In particular

$$\overline{\pi}_{t,1} = \mathbf{w}_t' \cdot diag(\mathbf{v}) \cdot \left(\mathbf{y}_{t+1}^o - \mathbf{y}_t^o\right) - \mathbf{c}_t' \cdot \left|\mathbf{w}_t - \mathbf{w}_{t-1}\right|$$
$$\mathbf{v}_t = \left(v_{1,t}, \dots, v_{10,t}\right)'$$

where:

– $v_{M,t}$ is the present value at the beginning of day t of one basis point of the fixed rate of the OIS whose life is $\left[t, t + M \cdot \Delta^{-1}\right]$; note that $y_{M,t}^o$ is measured in basis points;
– $\left(y_{M,t+1}^o - y_{M,t}^o\right) \cdot v_{M,t} \cdot w_{M,t}$ approximates $\left(y_{M-\Delta,t+1}^o \cdot v_{M-\Delta,t+1} - y_{M,t}^o \cdot v_{M,t}\right) \cdot w_{M,t}$.

The profit measure $\overline{\pi}_{t,1}$ marks the OIS contract to market each day, but it involves $v_{M,t}$ which depends on the OIS valuation model. To avoid this model-dependence, we focus on the model-free profit measure $\pi_{t,1}$. Anyway the two profit measures $\pi_{t,1}$ and $\overline{\pi}_{t,1}$ tend to give similar conclusions about the tested trading strategies. The profit measure $\pi_{t,1}$ can use the following approximation

$$\left(y_{M,t+1}^o - y_{M,t}^o\right) \cdot M \simeq \mathbf{e}_{M,N}' \cdot \left(\frac{\partial \mathbf{y}_t}{\partial \mathbf{x}_t'} \cdot \left(\mathbf{x}_{t+1} - \mathbf{x}_t\right) + \boldsymbol{\varepsilon}_{t+1} - \boldsymbol{\varepsilon}_t\right) \cdot M.$$

$\mathbf{e}_{M,N}$ is the Mth column of the $N \times N$ identity matrix $\mathbf{I}_N$; $\frac{\partial \mathbf{y}_t}{\partial \mathbf{x}_t'}$ is the $N \times n$ matrix of first derivatives (deltas) at time t; $N = 10$ is the longest OIS maturity measured in years. This is the approximation that inspires the delta-hedged strategies of this paper.

2.4. Overall performance measures

At the end of the sample, i.e. at the end of day T, total cumulative portfolio profit is computed. We compute two overall performance measures:

– the cumulative daily profit until T, which is

$$\sum_{t=1}^{T-1} \pi_{t,1}; \tag{3}$$

– the Realised Information Ratio (RIR) until T, which is

$$\theta_{t,T} = \frac{\frac{1}{T-1} \sum_{t=1}^{T-1} \pi_{t,1}}{\sqrt{\frac{1}{T-1} \sum_{t=1}^{T-1} \left(\pi_{t,1} - \frac{1}{T-1} \sum_{t=1}^{T-1} \pi_{t,1}\right)^2}}.$$

$\theta_{t,T}$ is the ratio between the sample average daily profit and the sample standard deviation of daily profit. As shown below, some strategies deliver high cumulative profit, but are not attractive because they deliver low RIR.

3. The portfolio strategies

We follow arbitrage portfolio strategies that maximise the ex-ante Information Ratio, referred to as IR, each day. Let $\theta_{\mathbf{w}_t}$ denote IR at the beginning of day t. Then

$$\theta_{\mathbf{w}_t} = \frac{\mathbf{w}_t' \cdot diag(\mathbf{M}) \cdot \mathbf{m}_{t,h} - \mathbf{c}_t' \cdot diag(\mathbf{M}) \cdot \left|\mathbf{w}_t - \mathbf{w}_{t-1}\right|}{\sqrt{\mathbf{w}_t' \cdot diag(\mathbf{M}) \cdot \mathbf{\Sigma}_{t,h} \cdot diag(\mathbf{M}) \cdot \mathbf{w}_t}}.$$

$\mathbf{w}_t$ is the vector of portfolio weights during the portfolio holding period $[t, t+h]$. Changing the portfolio weights at the beginning of day t incurs transaction costs equal to $\left|\mathbf{w}_t' - \mathbf{w}_{t-1}'\right| \cdot \mathbf{c}_t$, as the weights are updated each day. Let $\mathbf{w}_t^*$ be the portfolio weights that maximise $\theta_{\mathbf{w}_t}$, so that $\theta_{\mathbf{w}_t^*} \geq \theta_{\mathbf{w}_t}$ for all $\mathbf{w}_t$. Since we set $h = 1$, $\theta_{\mathbf{w}_t^*}$ and $\theta_{\mathbf{w}_t}$ assume a holding period of one day. We only open the portfolio position if $\theta_{\mathbf{w}_t^*} > \overline{\theta}_o$ and close the open position if $\theta_{\mathbf{w}_t^*} \leq \overline{\theta}_c$. Thus $\overline{\theta}_o$ and $\overline{\theta}_c$ are cutoffs for opening and closing a portfolio position. The portfolio strategy we follow is:

– if $\mathbf{w}_{t-1} = \mathbf{0}_{N\times 1}$ and $\theta_{\mathbf{w}_t^*} \leq \overline{\theta}_o$, then $\mathbf{w}_t = \mathbf{0}_{N\times 1}$;
– if $\mathbf{w}_{t-1} = \mathbf{0}_{N\times 1}$ and $\theta_{\mathbf{w}_t^*} > \overline{\theta}_o$, then $\mathbf{w}_t = \mathbf{w}_t^*$;
– if $\mathbf{w}_{t-1} \neq \mathbf{0}_{N\times 1}$ and $\theta_{\mathbf{w}_t^*} \leq \overline{\theta}_c$, then $\mathbf{w}_t = \mathbf{0}_{N\times 1}$;
– if $\mathbf{w}_{t-1} \neq \mathbf{0}_{N\times 1}$ and $\theta_{\mathbf{w}_t^*} > \overline{\theta}_c$, then $\mathbf{w}_t = \mathbf{w}_t^*$.

Recall that N is the number of OIS maturities in our sample. At the beginning of each trading day t, we close the open position if $\theta_{\mathbf{w}_t^*}$ is not higher than $\overline{\theta}_c$, and only if $\theta_{\mathbf{w}_t^*}$ is higher than $\overline{\theta}_o$ do we open or keep open the portfolio position. Keeping the position open entails rebalancing the portfolio weights so as to maximise $\theta_{\mathbf{w}_t}$ at the beginning of each trading day t. The empirical tests below assume $\overline{\theta}_o = \overline{\theta}_c = 0.2$. If the cutoffs $\overline{\theta}_o$, $\overline{\theta}_c$ were not positive, the portfolio position would be opened or be kept open even as it is expected to incur losses. If $\overline{\theta}_o = \overline{\theta}_c$ and $\mathbf{c}_t = \mathbf{0}_{N\times 1}$ transaction costs are absent and the portfolio strategy reduces to:

– $\mathbf{w}_t = \mathbf{w}_t^*$ if $\theta_{\mathbf{w}_t^*} > \overline{\theta}_o$ and $\mathbf{w}_t = \mathbf{0}_{N\times 1}$ if $\theta_{\mathbf{w}_t^*} \leq \overline{\theta}_o$.

3.1. The portfolio weights that maximise IR subject to delta-hedging

We determine optimal portfolio weights that maximise the ex-ante IR $\theta_{\mathbf{w}_t}$ for the one day holding period, while delta-hedging portfolio risk due to changes in the factors $\mathbf{x}_t$. To do so we maximise the Lagrangian function

$$L = \theta_{\mathbf{w}_t} + \mathbf{l}' \cdot \left(\frac{\partial \mathbf{y}_t'}{\partial \mathbf{x}_t} \cdot diag(\mathbf{M}) \cdot \mathbf{w}_t - \mathbf{0}_{n\times 1}\right).$$

$\mathbf{l}$ is a $n \times 1$ vector of Lagrange parameters. $\frac{\partial \mathbf{y}_t'}{\partial \mathbf{x}_t}$ is an $n \times N$ matrix of deltas. $\mathbf{0}_{n\times 1}$ is a $n \times 1$ vector of zeros. Recall that n is the number of factors in $\mathbf{x}_t$. We maximise L for each trading day t and there are 863 trading days in our sample.

Maximising L with respect to $\mathbf{w}_t$ is a challenge because of the term that measures transaction costs, i.e. $\mathbf{c}_t' \cdot \left|\mathbf{w}_t - \mathbf{w}_{t-1}\right|$. Note that $\left|\mathbf{w}_t - \mathbf{w}_{t-1}\right| = \left(\left|\pm\left(w_{1,t} - w_{1,t-1}\right)\right|, \ldots, \left|\pm\left(w_{N,t} - w_{N,t-1}\right)\right|\right)$ and $N = 10$ in our sample. Thus $\left|\mathbf{w}_t - \mathbf{w}_{t-1}\right|$ is identical for $2^{10} = 1024$ possible vectors $\left(\mathbf{w}_t - \mathbf{w}_{t-1}\right)$ each of which differs only in the sign of each element. We maximise L over all these 1024 possible vectors by enumeration. Then the optimal portfolio weights $\mathbf{w}_t^*$ are those that maximise L across all these 1024 possible vectors. We can write the maximum of L as $L^* = \max(\mathbf{L})$ where $\mathbf{L} = \left(L_1, L_2, \ldots, L_{1024}\right)'$. We compute L_p for $p = 1, 2, \ldots, 1024$ by setting

$$\begin{aligned}
\frac{\partial L_p}{\partial \mathbf{u}_t} &= \frac{\partial \theta}{\partial \mathbf{u}_t} + \frac{\partial\left(\mathbf{l}' \frac{\partial \mathbf{y}_t'}{\partial \mathbf{x}_t} diag(\mathbf{M})\, \mathbf{u}_t\right)}{\partial \mathbf{u}_t} \\
&= \frac{\overline{\mathbf{m}}_{t,h} - \overline{\mathbf{c}}_t}{\sqrt{\mathbf{u}_t' \overline{\mathbf{\Sigma}}_{t,h} \mathbf{u}_t}} - \frac{1}{2}\left(\mathbf{u}_t'\left(\overline{\mathbf{m}}_{t,h} - \overline{\mathbf{c}}_t\right) + \mathbf{w}_{t-1}' \overline{\mathbf{c}}_t\right)\left(\mathbf{u}_t' \overline{\mathbf{\Sigma}}_{t,h} \mathbf{u}_t\right)^{-\frac{3}{2}} 2\overline{\mathbf{\Sigma}}_{t,h} \mathbf{u}_t + diag(\mathbf{M}) \frac{\partial \mathbf{y}}{\partial \mathbf{x}_t'} \mathbf{l} = \mathbf{0}_{N\times 1}
\end{aligned}$$

$$\overline{\mathbf{m}}_{t,h} = diag(\mathbf{M}) \cdot \mathbf{m}_{t,h}$$
$$\overline{\mathbf{c}}_t = diag(\mathbf{M}) \cdot \mathbf{c}_t$$
$$\overline{\mathbf{\Sigma}}_{t,h} = diag(\mathbf{M}) \cdot \mathbf{\Sigma}_{t,h} \cdot diag(\mathbf{M})$$

which implies

$$\frac{\partial \theta}{\partial \mathbf{u}_t} + \frac{\partial\left(\mathbf{l}' \frac{\partial \mathbf{y}_t'}{\partial \mathbf{x}_t} \mathbf{u}_t\right)}{\partial \mathbf{u}_t} = \overline{\mathbf{m}}_{t,h} - \overline{\mathbf{c}}_t - \lambda_c \overline{\mathbf{\Sigma}}_{t,h} \mathbf{u}_t + diag(\mathbf{M}) \cdot \frac{\partial \mathbf{y}}{\partial \mathbf{x}_t'} \mathbf{l} = \mathbf{0}_{N\times 1}$$

$$\lambda_c = \left(\mathbf{u}_t'\left(\overline{\mathbf{m}}_{t,h} - \overline{\mathbf{c}}_t\right) + \mathbf{w}_{t-1}' \overline{\mathbf{c}}_t\right) \cdot \left(\mathbf{u}_t' \overline{\mathbf{\Sigma}}_{t,h} \mathbf{u}_t\right)^{-1}$$

$$\mathbf{u}_t = \left(\lambda_c \overline{\mathbf{\Sigma}}_{t,h}\right)^{-1} \cdot \left(\overline{\mathbf{m}}_{t,h} - \overline{\mathbf{c}}_t + diag(\mathbf{M}) \cdot \frac{\partial \mathbf{y}}{\partial \mathbf{x}_t'} \mathbf{l}\right)$$

$$\mathbf{w}_{p,t}^* = \mathbf{u}_t \cdot \frac{1}{\mathbf{u}_t' \cdot \boldsymbol{\iota}_N}$$

where $\boldsymbol{\iota}_N$ is an $N \times 1$ vector of ones. To find $\mathbf{l}$ we impose the constraints $\frac{\partial \mathbf{y}_t'}{\partial \mathbf{x}_t} \cdot diag(\mathbf{M}) \cdot \mathbf{u}_t = \mathbf{0}_{n\times 1}$, which imply

$$\frac{\partial \mathbf{y}_t'}{\partial \mathbf{x}_t} \cdot diag(\mathbf{M}) \cdot \left(\lambda_c \overline{\boldsymbol{\Sigma}}_{t,h}\right)^{-1} \cdot \left(\overline{\mathbf{m}}_{t,h} - \overline{\mathbf{c}}_t + diag(\mathbf{M}) \cdot \frac{\partial \mathbf{y}}{\partial \mathbf{x}_t'} \cdot \mathbf{l}\right) = \mathbf{0}_{n\times 1}$$

and

$$\mathbf{l} = -\left(\frac{\partial \mathbf{y}_t'}{\partial \mathbf{x}_t} \cdot diag(\mathbf{M}) \cdot \overline{\boldsymbol{\Sigma}}_{t,h}^{-1} \cdot diag(\mathbf{M}) \cdot \frac{\partial \mathbf{y}}{\partial \mathbf{x}_t'}\right)^{-1} \cdot \frac{\partial \mathbf{y}_t'}{\partial \mathbf{x}_t} \cdot diag(\mathbf{M}) \cdot \overline{\boldsymbol{\Sigma}}_{t,h}^{-1}\left(\overline{\mathbf{m}}_{t,h} - \overline{\mathbf{c}}_t\right).$$

Then $\mathbf{l}$ can substituted into the formula for $\mathbf{u}_t$ to determine $\mathbf{w}_{p,t}^*$, which are the portfolio weights that maximise L_p. Then $\theta_{\mathbf{w}_{p,t}^*}$ can be computed. Recall that the effective bid–ask spreads $\mathbf{c}_t$ measure the differences between the trade price and the mid price. The higher $\mathbf{c}_t$ are, the lower $\theta_{\mathbf{w}_{p,t}^*}$ is, because daily portfolio rebalancing is more costly. As stated above we assume $\alpha = 0, 1/2, 1/4, 1/8$, and we test two strategy types:

– Sc(α), which determines $L^*, \theta_{\mathbf{w}_t^*}, \mathbf{w}_t^*$ assuming $\mathbf{c}_t = \alpha \cdot \mathbf{s}_t$ as just described, thus taking into account the bid–ask spread transaction cost of rebalancing the portfolio; this optimisation is more burdensome, but more accurate than that of S(α);

– S(α), which maximises L to determine $\mathbf{w}_t^*$ while assuming $\mathbf{c}_t = \mathbf{0}_{N\times 1}$, thus avoiding the calculation of L_p for $p = 1, \ldots, 1024$; after $\mathbf{w}_t^*$ has been so determined, in S(α) we compute $\theta_{\mathbf{w}_t^*}$ assuming $\mathbf{c}_t = \alpha \cdot \mathbf{s}_t$; S($\alpha$) is much quicker to compute than Sc(α), but its optimisation is less accurate; when $\alpha = 0$, S(α) and Sc(α) coincide.

3.2. Calibration

All pricing models parameters $\boldsymbol{\mu}, \boldsymbol{\kappa}, \boldsymbol{\sigma}$ are calibrated by minimising squared pricing errors

$$\sum_{t=1}^{T} \left(\mathbf{y}_t^o - \mathbf{y}_t\right)' \cdot \left(\mathbf{y}_t^o - \mathbf{y}_t\right)$$

with respect to the said parameters and to the latent factors. Thus calibration also provides estimates of the latent factors $\mathbf{x}_t$ for $t = 1, \ldots, T$. T is the number of trading days in the sample. For a given set of parameter values $\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\kappa}}, \hat{\boldsymbol{\sigma}}$, we compute the latent factors for each t as

$$\hat{\mathbf{x}}_t = \left(\mathbf{Q}' \cdot \mathbf{Q}\right)^{-1} \cdot \mathbf{Q}' \cdot \mathbf{q} = \arg\min_{\mathbf{x}_t} \left(\mathbf{q} - \mathbf{Q} \cdot \mathbf{x}_t\right)' \left(\mathbf{q} - \mathbf{Q} \cdot \mathbf{x}_t\right)$$
$$\mathbf{Q} = \left[\frac{\partial \mathbf{y}_t}{\partial \mathbf{x}_t'}\right]_{\mathbf{x}_t = \mathbf{x}_{t-1}}$$
$$\mathbf{y}_t = \mathbf{y}_t\left(\mathbf{x}_t\right)$$
$$\mathbf{q} = \mathbf{y}_t^o - \mathbf{y}_t\left(\mathbf{x}_{t-1}\right) + \mathbf{Q} \cdot \mathbf{x}_{t-1}.$$

This calculation of $\hat{\mathbf{x}}_t$ uses the linear approximation $\mathbf{y}_t^o \simeq \mathbf{y}_t\left(\mathbf{x}_{t-1}\right) + \mathbf{Q} \cdot \left(\mathbf{x}_t - \mathbf{x}_{t-1}\right)$. This calibration resembles the non-linear least squares regression estimation in Ang and Longstaff (2013), among others, but differs in that no assumption about the pricing errors $\left(\mathbf{y}_t^o - \mathbf{y}_t\right)$ is needed. In particular the pricing errors need not be homoscedastic and uncorrelated in the cross section and in the time series. The evidence shows that for all tested models pricing errors can be correlated in the cross section and in the time series.

4. Empirical tests

4.1. Data and estimation

The sample data consists of 863 daily observations obtained from Refinitiv Workspace:

– Estr OIS bid–ask end-of-day prices for maturities of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 years, for each trading day over the period 2/10/2019 to 20/1/2023;

– Estr rate daily closings computed by the ECB for the same days.

The sample start date is 2/10/2019, because the ECB started publishing Estr on that day. Eonia, Estr's predecessor, was computed in a different way and, because of this, was often more "spiky" and volatile than Estr. The Eonia regime is now over, and empirical results covering both the Eonia and Estr regimes would have been more difficult to interpret. Thus the paper only uses OIS that reference Estr, not OIS that reference Eonia. Fig. 1 displays the Estr rate and OIS rates in the sample.

Estr is persistently a few basis points, often around 10 basis points, lower than the ECB deposit facility rate, since not all parties lending at the Estr rate can deposit at the ECB. Some such parties are not banks and lend to banks at the Estr rate. The banks that borrow at Estr can then deposit the borrowed money at the ECB at the deposit facility rate. While Estr exhibits "step changes" often referred to as jumps, it does not exhibit spikes, i.e. jumps that are immediately reversed. The method the ECB uses to compute Estr has been designed to avoid such spikes. Instead in the past decade Eonia sometimes exhibited spikes.

All pricing models are calibrated to Estr and Ester OIS rates as explained above. However model A4-E is calibrated to Estr OIS rates only, and not to Estr. To ensure that the tested trading strategies are mostly out of sample ones, all model parameters are calibrated using rolling windows of 200 days as explained in Appendix. All trading strategies use delta-hedged market-neutral portfolios, as explained above, since portfolio profitability tends to be too volatile in the absence of delta hedging. Delta hedging hedges risk due to the latent factors driving Estr, but not risk due to pricing errors.

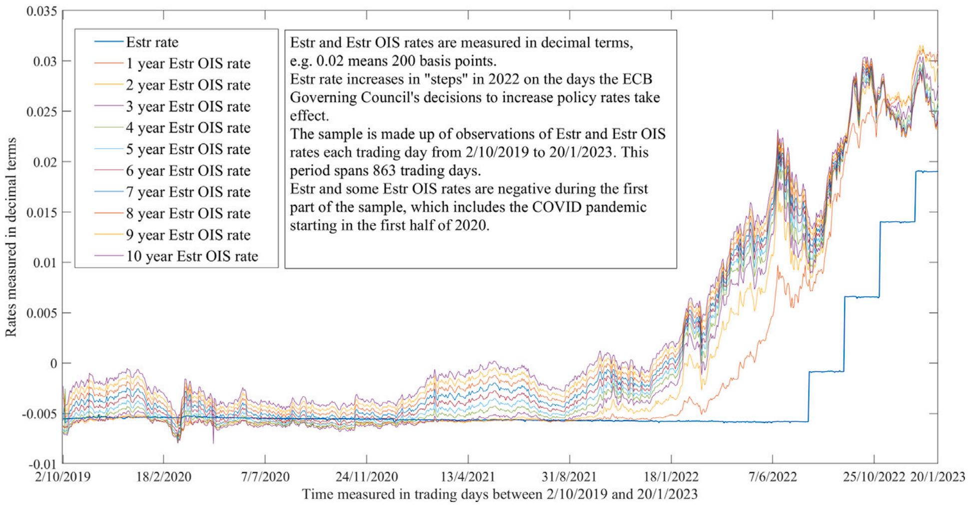

Fig. 1. Daily observations of Euro Short Term Rate (Estr) and Estr Overnight Index Swaps (Estr OIS) rates in the sample.

4.2. Empirical results

Table 1 summarises the calibrated models. Models A5-HW, A4-HW and A3-HW are respectively five, four and three factor models in which the HW conditions of Eq. (2) are imposed. All models use Estr in calibration, assume that all OIS are observed with errors and assume the HW conditions in Eq. (2), except for:

– A4; this is a four factor model that assumes the conditions in Eq. (1) in place of the conditions in Eq. (2);

– A4-E; this is the same as A4, but does not use Estr in calibration;

– A4-Ee; this is the same as A4-E; like A4-E, it does not use Estr in calibration; unlike A4-E, it assumes that 1, 2, 9, 10 year OIS rates are observed without errors; for this model $\hat{\mathbf{x}}_t$ is computed with non-linear least squares as shown above, but using only OIS rates of the 1, 2, 9, 10 year maturities, rather than using all OIS rates; in this way A4-Ee can almost perfectly fit the OIS rates of the said four maturities.

Table 1 reports Root Mean Squared pricing Errors (RMSE) and calibrated parameters for each model. A pricing error is the differences between an observed OIS rate and the OIS rate predicted by a model. RMSE are about 2 to 3 basis points across models. RMSE are indicative of models relative performance.

RMSE show that A4 and A4-HW fit Estr OIS similarly. A4-HW is subject to conditions (2), which assume that Estr can only change when ECB policy rates change, while A4 is subject to conditions (1), which assume that Estr can change each day. In fact Estr hardly changes on days other than those on which ECB policy rates change.

RMSE are higher for A4-Ee, which assumes no observation error for four OIS maturities, than for the other four factor models. The reason is that imposing no observation errors for some OIS maturities reduces the freedom of the model to fit OIS rates of other maturities.

Then RMSE show that A4-E, which is subject to conditions (1) but is not calibrated to Estr, fits OIS rates more closely than A4, which is subject to conditions (1) and is calibrated to Estr as well as to OIS rates. The reason is that including Estr in calibration adds one more calibration requirement.

The RMSE show that neither removing the HW conditions in Eqs. (2) nor removing Estr in calibration hinders the fit of the four factor model to OIS rates. In fact A4-E reports the lowest RMSE, closely followed by A5-HW. Instead A3-HW clearly worst fits OIS rates. Three factors seem too few.

Table 1 also reports the mean, standard deviation and first order auto-correlation of the end-of-day bid–ask spreads for each OIS maturity. The said spreads are relatively high, in that they average around 4 basis points, but their low one-day-auto-correlation coefficients show that they are not very persistent over successive days.

4.3. Trading strategies profitability and bid–ask spreads

Figs. 2, 3, 4 and Table 2 show the profitability of trading strategies that use the calibrated models. All the tested trading strategies have a normalised net notional exposure of 1 or −1, where 1 is the notional amount of one Estr OIS contract. Such notional amount is also the unit that measures the nominal daily profit $\pi_{t,1}$ of the strategies. Table 2 shows four profitability measures for each model and strategy, and summarises all tests of models and strategies.

The profitability of the S(α) strategies described above tends to rise considerably when portfolio positions are opened or kept open only if the maximised ex-ante IR $\theta_{\mathbf{w}_t^*}$ is higher than set positive cutoffs $\overline{\theta}_o, \overline{\theta}_c$. Removing these positive cutoffs leads to huge

Table 1
Summary of models and bid–ask spreads.

Panel A: RMSE (root mean squared errors in decimal terms) for each model

RMSE	Models					
OIS maturities	A4-Ee	A4-E	A4	A4-HW	A5-HW	A3-HW
1 year	0.00021	0.00022	0.00027	0.00028	0.00023	0.00140
2 years	0.00023	0.00024	0.00031	0.00031	0.00027	0.00089
3 years	0.00053	0.00025	0.00027	0.00028	0.00025	0.00039
4 years	0.00071	0.00026	0.00027	0.00027	0.00027	0.00053
5 years	0.00066	0.00023	0.00025	0.00025	0.00023	0.00062
6 years	0.00054	0.00022	0.00024	0.00024	0.00022	0.00057
7 years	0.00038	0.00020	0.00021	0.00021	0.00020	0.00043
8 years	0.00019	0.00013	0.00013	0.00013	0.00013	0.00019
9 years	0.00010	0.00011	0.00013	0.00013	0.00011	0.00029
10 years	0.00008	0.00010	0.00015	0.00015	0.00010	0.00065
All maturities	0.00043	0.00020	0.00023	0.00023	0.00021	0.00068

LEGEND: This Panel reports RMSE for each model and each OIS maturity from 1 to 10 years.
“OIS” means “Overnight Index Swaps”. OIS maturity is the time duration of the OIS contract.
The row “All maturities” reports RMSE computed across all maturities.
RMSE are expressed in decimal terms: for example 0.00043 is 4.3 basis points.
RMSE are computed over all rolling windows used in calibration as explained in the Appendix.

Panel B: Model features for each model

Models features	Models					
Features	A4-Ee	A4-E	A4	A4-HW	A5-HW	A3-HW
HW conditions?	No	No	No	Yes	Yes	Yes
Observations errors for all OIS’s?	No	Yes	Yes	Yes	Yes	Yes
Estr in calibration?	No	No	Yes	Yes	Yes	Yes

LEGEND: This Panel highlights the features of each tested pricing model.
HW conditions are given in Eqs. (2) and imposed on models suffixed with “-HW”.
“Observation errors for all OIS’s”: yes means that OIS of all maturities are assumed observed with errors; no means that some OIS maturities are assumed observed with no errors.
“Estr” means “Euro Short Term Rate”.
“Estr in calibration”: yes means that each day the model is calibrated to both Estr and OIS rates; no means that each day the model is calibrated only to OIS rates.

Panel C: Calibrated model parameters of each model

Parameter calibration	Models					
Parameters	A4-Ee	A4-E	A4	A4-HW	A5-HW	A3-HW
$\mu 1$	0.0011	0.0023	0.0039	0.0003	0.0024	0.0002
$\sigma 1$	0.0000	0.0000	0.0216	0.0000	0.0026	0.0000
$\sigma 2$	0.0118	0.0000	0.0041	0.0000	0.0222	0.0215
$\sigma 3$	0.0858	0.0039	0.0005	0.1066	0.0066	0.0000
$\sigma 4$	0.5894	0.5962	0.0003	0.1498	0.0639	
$\sigma 5$					0.0002	
$\kappa 1$	0.0000	0.2713	2.3754	0.4206	0.1031	0.0000
$\kappa 2$	0.0000	0.0000	0.0000	0.0000	1.1545	1.4037
$\kappa 3$	0.0000	2.6278	2.3761	2.2981	0.0516	0.1613
$\kappa 4$	2.0051	2.5427	0.3933	0.2494	1.5293	
$\kappa 5$					0.1755	

LEGEND: The parameters reported in this Panel are calibrated to the whole sample.

Panel D: bid–ask spread statistics (in decimal terms) for Estr OIS’s of all maturities

Bid–ask spreads	Models		
OIS maturities	mean	std dev	One day autocorrelation
1 year	0.00019	0.00019	0.45522
2 years	0.00020	0.00018	0.36114
3 years	0.00046	0.00009	0.41277
4 years	0.00046	0.00010	0.31466
5 years	0.00045	0.00009	0.27964

(*continued on next page*)

Table 1 (*continued*).

6 years	0.00045	0.00009	0.38117
7 years	0.00045	0.00009	0.43996
8 years	0.00045	0.00009	0.41388
9 years	0.00046	0.00008	0.27475
10 years	0.00046	0.00009	0.36664

LEGEND: Bid–ask spread is the difference between the ask OIS rate and bid OIS rate.
The reported bid–ask spread statistics are computed across the whole sample
and expressed in decimal terms, e.g. 0.0002 is 2 basis points.
“mean” and “std dev” are the mean and standard deviation of the spread for each maturity.
“one-day-auto-correlation” is the correlation of the bid–ask spreads of two consecutive trading days.

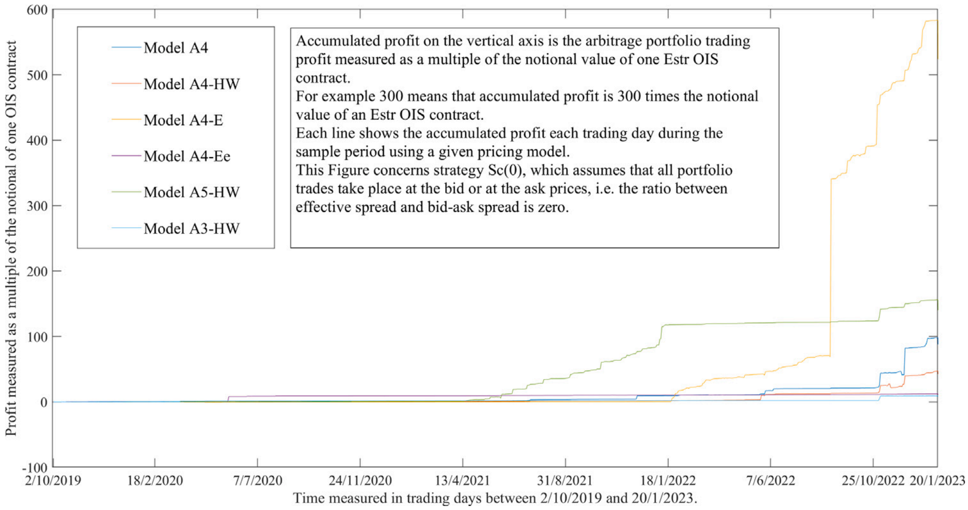

Fig. 2. Daily cumulative profit of strategy Sc(0) for each model over the sample period.

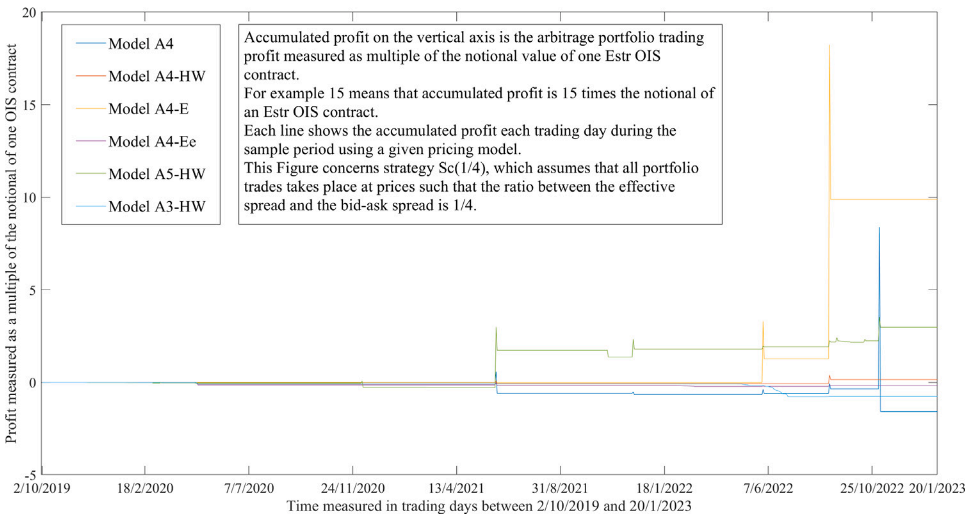

Fig. 3. Daily cumulative profit of strategy Sc(1/4) for each model over the sample period.

losses with all models, since portfolio positions expected to generate losses would be opened or kept. All the strategies we report assume the cutoffs $\overline{\theta}_c = \overline{\theta}_o = 0.2$. Higher cutoffs increase the chance that portfolio positions be profitable, but also reduce the number of profit opportunities. The profitability of S(α) strategies is sensitive to the said cutoffs, while that of Sc(α) strategies is much less.

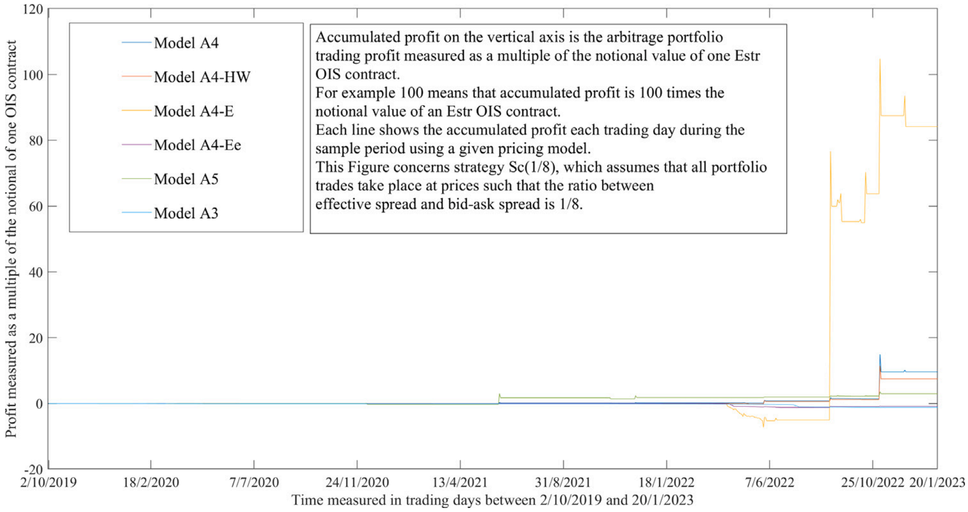

Fig. 4. Daily cumulative profit of strategy Sc(1/8) over the sample period.

Table 2
Summary of information ratio (IR) and profit for each model and for each strategy.

Strategies	S(0)	S(1/2)	Sc(1/2)	S(1/4)	Sc(1/4)	S(1/8)	Sc(1/8)
Model A4							
Cumulative daily profit	98.31	No trade	−0.01	0.10	−1.57	9.80	9.62
Average ex-ante IR	1.54	−21.04	−8.85	−9.41	−3.99	−4.10	−1.39
Realised IR (daily)	0.07	No trade	−0.02	0.01	−0.00	0.02	0.02
Realised IR (annualised)	1.15	No trade	−0.25	0.10	−0.06	0.30	0.36
Model A4-HW							
Cumulative daily profit	47.47	No trade	−0.01	No trade	0.16	2.98	7.45
Average ex-ante IR	1.50	−20.59	−8.86	−9.41	−4.07	−3.75	−1.56
Realised IR (daily)	0.09	No trade	−0.02	No trade	0.01	0.02	0.02
Realised IR (annualised)	1.37	No trade	−0.24	No trade	0.16	0.29	0.37
Model A4-E							
Average margin	0.91	0.00	0.00	0.05	0.01	0.06	0.27
Cumulative daily profit	583.50	No trade	No trade	**26,540**	9.89	**338.04**	84.17
Average ex-ante IR	1.92	−36.43	−10.76	−212.27	−4.80	−6.08	−1.60
Realised IR (daily)	0.07	No trade	No trade	0.01	0.02	0.03	0.03
Realised IR (annualised)	1.14	No trade	No trade	0.16	0.28	0.41	0.48
Model A4-Ee							
Cumulative daily profit	12.38	−0.07	−0.01	−0.04	−0.18	−0.35	−0.84
Average ex-ante IR	1.79	−20.15	−9.20	−10.02	−4.07	−4.07	−1.41
Realised IR (daily)	0.05	−0.05	−0.03	−0.01	−0.02	−0.04	−0.04
Realised IR (annualised)	0.85	−0.78	−0.50	−0.15	−0.40	−0.57	−0.68
Model A5-HW							
Cumulative daily profit	156.29	No trade	No trade	−0.51	2.98	5.76	2.98
Average ex-ante IR	1.37	−23.01	−9.18	−10.50	−1.72	−4.47	−1.72
Realised IR (daily)	0.18	No trade	No trade	−0.00	0.03	0.02	0.03
Realised IR (annualised)	2.83	No trade	No trade	−0.07	0.41	0.34	0.41

(*continued on next page*)

When the effective bid–ask spread is greater than zero, strategies Sc(α) and S(α) differ, and Sc(α) is expected to fare better, but need not always do. While maximised ex-ante IR $\theta_{\mathbf{w}_t^*}$ is higher for Sc(α) than for S(α), other things equal, realised IR $\theta_{t,T}$ is often, but not always, higher for Sc(α) than for S(α). The reason is that pricing errors may evolve in unexpected ways.

For each model Figs. 2, 3, 4 report cumulative daily profit computed as in Eq. (3) for strategies S(α) and Sc(α) for different sizes of the effective bid–ask spread as determined by α. Fig. 2 concerns S(0) and Sc(0), which are the most optimistic strategies for the investor, as they assume that the effective bid–ask spread is zero at all times, i.e. $\alpha = 0$, so that all trades occur at the mid-price and rebalancing the portfolio costs nothing. Sc(0) and S(0) are effectively the same and are highly profitable for all models. Profitability can be staggering for some models, because portfolio positions are extremely aggressive since it costs nothing to trade. Profitability of Sc(0) and S(0) is very sensitive to the cutoffs $\overline{\theta}_o, \overline{\theta}_c$, and setting $\overline{\theta}_o = \overline{\theta}_c = 0.2$ proved a suitable choice.

Table 2 (*continued*).

Strategies	S(0)	S(1/2)	Sc(1/2)	S(1/4)	Sc(1/4)	S(1/8)	Sc(1/8)
Model A3-HW							
Cumulative daily profit	9.36	−0.00	−0.00	−1.75	−0.76	−0.66	−1.15
Average ex-ante IR	1.84	−18.62	−8.63	−9.20	−3.63	−3.29	−0.90
Realised IR (daily)	0.06	−0.04	−0.05	−0.06	−0.14	−0.13	−0.11
Realised IR (annualised)	1.00	−0.67	−0.77	−0.99	−2.19	−2.13	−1.74

LEGEND: Each Panel in this Table refers to a different model. Each column refers to a different strategy.
IR is the information ratio.
Strategy S(0) assumes that all trades are at the mid-price, i.e. at the mid OIS rate, as if the market was frictionless.
S and Sc strategies are described in the text. Sc strategies tend to be more profitable than S strategies, because they maximise ex-ante IR taking bid–ask spreads into account, while S strategies maximise ex-ante IR without taking bid–ask spreads into account.
The effective spread is the absolute value of the difference between mid price and trade price.
Strategies S(1/2) and Sc(1/2) assume that the effective spread is half of the bid–ask spread. This means that the investor always buys at the ask OIS rate and always sells at the bid OIS rate.
Strategies S(1/4) and Sc(1/4) assume that the ratio between effective spread and bid–ask spread is 1/4.
Strategies S(1/8) and Sc(1/8) assume that the ratio between effective spread and bid–ask spread is 1/8.
Cumulative daily profit is total profit of the strategy at the end of the sample. For example 9.36 means that such profit is 9.36 times the notional amount of one OIS contract.
Average ex-ante IR is ex-ante information ratio (IR) averaged over all trading days in the sample.
Realised IR (daily) is RIR as computed in Section 2.4.
Realised IR (annualised) is RIR times the square root of 260 (as there are about 260 trading days per year in the sample).
The cells of this Table that show "No trade" mean that for that model and strategy no portfolio position is opened in the sample period.
For model A4-E the row "Average margin" (highlighted) reports the average daily net margin requirement for each strategy. The text explains how "Average margin" is computed.

Strategies S(1/2) and Sc(1/2) assume the highest transaction costs for the investor, who only trades at the quoted bid and the ask prices, so that the effective spread is half of the bid–ask spread at all times, i.e. $\alpha = 1/2$. In this case no trading strategy seems attractive as all trading profitability is eliminated, as reported in Table 2. Sc(1/2) and Sc(1/2) are not appealing with any model and often lead to no trade in the whole sample, because maximised ex-ante IR $\theta_{\mathbf{w}_t^*}$ is each day lower than the cutoff $\overline{\theta}_o$. The cells of Table 2 that show "no trade" mean that, for the given model and strategy, no portfolio position is opened during the sample period due to lack of expected profitability.

Fig. 3 concerns S(1/4) and Sc(1/4), which assume that the effective spread is a quarter of the bid–ask spread. Fig. 4 concerns S(1/8) and Sc(1/8), which assume that the effective spread is one eighth of the bid–ask spread. As expected, trading strategies profitability tends to drop, but not always, as the effective bid–ask spread rises. Sc(1/8) and even Sc(1/4) can be quite profitable for some models, especially A5 and A4-E. Profitability tends to be very sensitive to the effective spread as determined by α. For example S(1/8) and Sc(1/8) are much less profitable than S(0) and Sc(0) and rely on just few upward jumps in cumulative profit. This is the case even for models A5 and A4-E, which are the most profitable models.

In Table 2 model A4-E shows an astronomic cumulative profit of 26,540 times the notional of one OIS contract under strategy S(1/4), which is even higher than the already very high cumulative profit under strategy S(1/8) for the same model. These profits are almost entirely made on day 759 of the sample, which is in the second half 2022, and are due to extremely aggressive portfolio weights on that day. For example S(1/4) implies portfolio weights that involve positions in millions or even tens of millions of OIS contracts on that day. Such large positions do not seem feasible in reality. Therefore the extreme cumulative profits for strategies S(1/4) and S(1/8) do not seem achievable in reality, although a fraction of such profits is achievable with less aggressive portfolio weights.

Overall Figs. 2, 3, 4 and Tables 1 and 2 make some clear points. When investors always trade at the mid-price, all models are profitable, but some models are much more than others. Affine models seem capable to detect mispriced OIS. However no model is profitable when investors can only trade at the bid or at the ask. Profitability tends to increase as the effective spread decreases, and is very sensitive to the effective spread.

4.4. Jumps and spikes in cumulative profit

The trading strategies occasionally show large "jumps" in cumulative daily profit. This is the case especially in Figs. 3 and 4. These jumps can be upward or downward and correspond to days when portfolio weights are extreme in absolute value, i.e. very "aggressive", often involving thousands of OIS contracts. Given the large size of the Estr OIS market, it seems feasible to take such "aggressive" positions. Aggressive weights can generate larger profit and are more likely when it costs nothing to rebalance the portfolio, as in strategies S(0) and Sc(0). Instead, when the effective bid–ask spread is greater than zero, it is costly to rebalance the portfolio, the portfolio weights that maximise ex-ante IR become less aggressive, and large jumps in profit are fewer.

A few of the model-strategy combinations in the Figures exhibit "spikes" in cumulative profit, i.e. upward jumps immediately followed by downward jumps. These downward jumps are due to exiting the portfolio position after pricing errors have corrected themselves. After such correction, ex-ante IR tends to fall below the cutoff and can even become negative. This triggers the closing of the portfolio position, which incurs transaction costs as the "exit" trades are costly. Profit upward jumps are achieved through extreme portfolio weights, and the transaction costs to exit such extreme positions are high, which causes profit downward jumps just after upward jumps. The exit cutoff $\overline{\theta}_c$ is set without regard to the transaction costs of exiting the portfolio position, but avoids the large expected losses implied by keeping the position when ex-ante IR becomes negative.

Figs. 2 to 4 show that upward jumps in profit have been more frequent since OIS rates began to rise in 2021. Strategies profitability appears to increase when rates are higher.

4.5. Trading profitability and model specification issues: HW conditions, Estr in calibration, no observation errors, the number of factors

Table 2 largely confirms the indications of the RMSE in Table 1 and provides various insights into the specification of models.

The models with the lowest RMSE in Table 1 are also the most profitable ones in Table 2, as they give the highest realised IR (RIR). These models are A4-E and A5.

A4 and A4-HW are similarly profitable, but not very profitable. The case for A4-HW seems stronger when the model is calibrated to both Estr and OIS rates. A4-HW, unlike A4, explicitly models Estr's jumps according to conditions (2) when the ECB policy rates change. Yet A4 and A4-HW fare similarly in our sample made up of 1 year to 10 year OIS. Explicitly modelling Estr's jumps may be more relevant to price short OIS maturities under one year, which are not in our sample.

Trading strategies tend to be more profitable for A4-E than for A4. This result is consistent with the lower RMSE for A4-E than for A4 in Table 1. A4-E is subject to conditions (1) and is calibrated to OIS rates but not to Estr. A4 is the same as A4-E except that it is calibrated to both OIS rates and to Estr, which reduces the ability of A4 to fit OIS rates, thus making it less profitable than A4-E.

Calibrating A4-Ee to OIS rates while assuming no observation errors for four OIS maturities introduces additional constraints in calibration. The result is that A4-Ee is less profitable than the other four factor models. This is consistent with the far higher RMSE in Table 1 for A4-Ee than for the other four factor models. Thus, when trading to exploit OIS mispricings, it seems detrimental to assume that some OIS rates are perfectly observed. This seems of interest since part of the literature that tests pricing models on the Treasury yield curve routinely assumes that some yields or combinations thereof are observed with no error, e.g. Joslin et al. (2011).

Also the number of model factors affects trading strategy profitability. This paper focuses on three to five factors, so that $n = 3, 4, 5$. More factors reduce the size of the pricing errors, because they give the pricing model more flexibility to match the term structure of OIS rates. However smaller pricing errors may not be desirable if due to the model over-fitting OIS rates. Then, more factors also entail more delta-hedging constraints in determining portfolio weights that maximise ex-ante IR. Table 2 shows support for trading strategies based on models with four or five factors rather than three factors. Three factor model A3-HW is clearly the least profitable model. Three factors do not seem enough. Again this conclusion is consistent with the RMSE in Table 1, which are highest for A3-HW.

The above tests are based on daily Estr OIS closing prices. Then the pricing errors across different OIS maturities may be due to non-synchronous prices, as closing prices may not be synchronous. To alleviate this concern the above strategies were also tested using intraday minute-by-minute prices on 2nd and 3rd February 2023. Again trading at the mid price was very profitable, but trading at the bid and ask eliminated all profit.

4.6. Margin requirements

We may scale the nominal profit of each strategy reported above by the capital committed to the strategy, but it is not obvious how to measure such capital, because initial and maintenance margin requirements for the Estr OIS positions are not easy to measure. The rules for the margin of net positions can be complicated, depend on the central clearing counterparty (CCP) and on intermediating brokers. Such rules can also change over time with the CCP's risk management policy.

In general initial margin tends to be higher for longer OIS maturities and therefore cannot simply be a fixed proportion of the net notional exposure, which is 1 or -1 in our strategies above, because such net exposure comprises OIS of different maturities. A possible approximation of capital to commit to margin is to set initial and maintenance margins equal to 10% of the absolute value of the algebraic sum of maturity weighted OIS notional positions. Thus margin would be 10% of the net maturity weighted notional position, regardless of whether such net position is long or short. In fact margin requirements are often the same whether a position is long or short. The said approximation of capital to commit to margin disregards how the market value of the portfolio position changes after the position has been opened. Disregarding the market value of the opened portfolio position seems to overstate margin capital needed, since such market value, even as approximated by accumulated nominal profit, tends to be positive and a positive portfolio value tends to reduce the maintenance margin requirement of the opened portfolio.

Table 2 shows the average daily net margin requirement so computed for model A4-E for each strategy. Strategy S(0) is the same as Sc(0) and requires the most capital, about 0.9 times the notional of one OIS on average each day. This is the most profitable strategy and the one that most often opens portfolio positions or keeps such positions open. Then Sc(1/8) requires capital of 0.26 times the notional of one OIS on average each day. All the other strategies require even less capital on average each day, since they are less profitable and open portfolio positions or keep such positions open much less frequently. These estimates show that relatively little capital is needed by each strategy to meet average daily margin requirements. What reduces the appeal of the trading strategies is not so much the capital to commit to margin, but the standard deviation of daily profit, which depresses the realised IR $\theta_{t,T}$ reported in Table 2. The said standard deviation is often high and highlights that the tested arbitrage portfolio strategies are by no means risk-free, even as each portfolio is delta-hedged. The reason is that OIS pricing errors can be volatile and evolve in unexpected ways.

Table 3
Auto-correlation coefficients (and associated p values) of pricing errors of consecutive days.

Models	A4-E		A5-HW		A3-HW		A4-HW		A4-Ee		A4	
OIS maturities	Coefficient	p value	Coefficient	p value	Coefficient	p value	Coefficient	p value	Coefficient	p value	Coefficient	p value
1 year	0.24	0.00	0.91	0.00	0.99	0.00	0.96	0.00	0.36	0.00	0.96	0.00
2 year	0.63	0.00	0.95	0.00	0.99	0.00	0.95	0.00	0.39	0.00	0.95	0.00
3 year	0.57	0.00	0.71	0.00	0.95	0.00	0.87	0.00	0.96	0.00	0.85	0.00
4 year	0.45	0.00	0.84	0.00	0.98	0.00	0.86	0.00	0.96	0.00	0.82	0.00
5 year	0.40	0.00	0.74	0.00	0.98	0.00	0.89	0.00	0.94	0.00	0.91	0.00
6 year	0.53	0.00	0.68	0.00	0.98	0.00	0.91	0.00	0.88	0.00	0.92	0.00
7 year	0.61	0.00	0.76	0.00	0.98	0.00	0.87	0.00	0.75	0.00	0.86	0.00
8 year	0.32	0.00	0.64	0.00	0.81	0.00	0.47	0.00	0.47	0.00	0.37	0.00
9 year	0.11	0.00	0.13	0.00	0.93	0.00	0.46	0.00	0.25	0.00	0.51	0.00
10 year	0.54	0.00	0.80	0.00	0.96	0.00	0.85	0.00	0.50	0.00	0.86	0.00
Average correlation	0.44		0.71		0.96		0.81		0.65		0.80	

LEGEND: This Table reports estimates of one day auto-correlation coefficients of pricing errors for each model.
The columns headed "coefficient" report the estimated auto-correlation coefficient for each model and each OIS maturity.
"OIS" means "Overnight Index Swaps". OIS maturity is the time duration of the OIS contract.
The columns headed "p value" report the p values of the tests of the null hypothesis of no auto-correlation. The lower the p value, the stronger is the evidence against the null hypothesis.
All p values clearly reject the hypothesis of no auto-correlation for each model and for each OIS maturity.
The row "Average correlation" averages the correlation coefficients of a model across all OIS maturities.
The estimates in this Table use the whole sample of OIS rates, but exclude the first 10 trading days of the sample.
The pricing errors this Table uses are computed using pricing models calibrated over rolling windows as explained in the Appendix.

4.7. Correlation of pricing errors

Term structure models in the literature typically assume white noise pricing errors. This assumption is convenient from a statistical point of view and is often deemed to reflect rational pricing by the market, as rational prices would imply zero pricing errors on average. However white noise pricing errors would imply trading opportunities that can consistently earn abnormal returns, a challenge to market efficiency. As Adrian et al. (2013) pointed out for a term structure model fitted to the US Treasury yield curve, white noise pricing errors imply negatively auto-correlated bond returns. Investors could consistently exploit such negative auto-correlation to reap abnormal returns. Yet US Treasury bond returns do not seem negatively auto-correlated. It seems puzzling that the literature testing term structure models assumes arbitrage free pricing models and at the same time white noise pricing errors. The latter seem at odds with marker efficiency.

The affine OIS pricing models in this paper produce pricing errors that are not white noise processes. The pricing errors are strongly positively auto-correlated at the daily frequency and also positively cross correlated across OIS maturities. Table 3 displays estimates of one-day-auto-correlation coefficients of pricing errors for each OIS maturity and for each pricing model. All estimated auto-correlation coefficients are positive, often close to 1, and all are significant as shown by the reported p values. The stronger the said auto-correlation, the more slowly the pricing errors revert toward zero, which reduces the profitability of trading strategies that expect such reversion toward zero. Instead white noise pricing errors imply negatively auto-correlated changes in OIS rates and quick reversion of pricing errors toward zero, which investors could profitably exploit.

Strongly auto-correlated pricing errors have two main implications. The first implication is that none of the tested pricing models can claim to be the "right" rational pricing model, assuming one such model exists. The second implication, which is quite consistent with efficiency of the Estr OIS market, is that the high arbitrage returns implied by white noise pricing errors do not seem available, even as arbitrage can still be profitable despite auto-correlated pricing errors. It is telling that the most profitable and best fitting models, namely A5-HW and A4-E, tend to be the ones whose pricing errors display the weakest auto-correlations in Table 3. However consistently profitable trading strategies do not seem easily available in the Estr OIS market to investors who can only ever trade at the bid or at the ask quotes.

4.8. Practical implications

The empirical evidence has a few practical implications.

Policy makers and investors who are to interpret signals from the young Estr OIS market, can regard such market as a quite efficient one, even as it appears less liquid than the Eonia OIS market because of its higher bid–ask spreads.

Arbitrageurs had better have bargaining power similar to that of market makers, when negotiating Estr OIS trades. If on average the effective spread at which they can trade is not less than a quarter of the bid–ask spread, arbitrage does not seem attractive. Arbitrage can be attractive when the effective spread is one eighth of the bid–ask spread, and more so when OIS rates rise above zero. Arguably bargaining power is more important than pricing models specifications. Then, if arbitrage portfolio positions are opened only when OIS mispricing is highly likely to be profitable, such positions are seldom opened.

The implications for financial engineers are to focus on four factor and five factor OIS pricing models, rather than three factor ones, and not to assume that some OIS rates are observed without errors. For pricing one year or longer maturity OIS, it does not seem necessary to use Estr in pricing models calibration or to explicitly model the "jumps" of Estr as ECB policy rates change.

5. Conclusion

Inspired by the market-neutral no-arbitrage swap trading strategies of Bali et al. (2009) and of Jarrow et al. (2019), this paper has tested new market-neutral trading strategies involving new swaps, namely Estr OIS. To link Estr OIS pricing to expected changes in Estr, this paper has also adapted the affine pricing model by Heidari and Wu (2010).

Standard no-arbitrage affine term structure models with three to five factors cannot fit Estr OIS rates in such a way that pricing errors are not auto-correlated. Pricing errors are strongly positively serially correlated and hamper the profitability of trading strategies that use such errors as signals of mispricing. An investor who could always trade at the mid-price can still realise extraordinary information ratios. An investor who always trades at bid and ask prices cannot profit. An investor who can "split" the bid–ask spread can profit from arbitrage. Overall these results show that the young Estr OIS market is quite efficient. Then affine models with four or five factors are more profitable than affine models with three factors. Moreover assuming that some OIS rates are observed without error hinders the profitability of models and strategies.

The tested affine models for pricing Estr OIS are quite standard, and future research may employ more sophisticated models, but finding one that can consistently profit in the Estr OIS market seems extremely difficult when trading at quoted bid and ask prices. This difficulty is mainly due to the large bid–ask spreads of Estr OIS, which average about 4 basis points in our sample. We note that Eonia OIS bid–ask spreads often averaged about 2 basis points in the past decade. The higher current bid–ask spreads are consistent with regulatory reforms, as explained in Duffie (2018), and possibly also with funding value adjustments, as explained in Andersen et al. (2019). As other literature has noted, safer intermediaries appear to be associated with less liquid financial markets.

CRediT authorship contribution statement

Marco Realdon: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Appendix. The rolling windows

To ensure that the tested trading strategies are mostly out of sample ones, all model parameters are calibrated using rolling windows as follows. A model is calibrated:

– to data from day 1 to day 200 of the sample; the calibrated parameters are used to compute OIS prices from day 1 to day 400 of the sample;

– to data from day 201 to 400 of the sample; the calibrated parameters are used to compute OIS prices from day 401 to day 600 of the sample;

– to data from day 401 to 600 of the sample; the calibrated parameters are used to compute OIS prices from day 601 to day 800 of the sample;

– to data from day 601 to 800 of the sample; the calibrated parameters are used to compute OIS prices from day 801 to day 863 of the sample.

During any given window, the pricing model uses parameter values estimated in the previous window, except in the first window, during which the pricing model uses parameters estimated in the first window itself. The parameters of the matrices $\mathbf{\Phi} = diag(\boldsymbol{\phi})$ and $\mathbf{\Sigma}_{t,1} = var_t\left[\epsilon_{t+1}\right]$ are recursively estimated through OLS each day using past errors observations $\Theta_t = \left\{\boldsymbol{\varepsilon}_1, \ldots, \boldsymbol{\varepsilon}_t\right\}$. We recall that $\boldsymbol{\varepsilon}_{t+1} = \mathbf{y}^o_{t+1} - \mathbf{y}_{t+1}$. In particular the matrix $\boldsymbol{\Sigma}_{t,1}$ is estimated as $\sum_{s=2}^{t} \frac{1}{t-1}\left(\boldsymbol{\varepsilon}_s - \boldsymbol{\Phi}\boldsymbol{\varepsilon}_{s-1}\right)\cdot\left(\boldsymbol{\varepsilon}_s - \boldsymbol{\Phi}\boldsymbol{\varepsilon}_{s-1}\right)'$.

References

Adrian, T., Crump, R.K., Moench, E., 2013. Pricing the term structure with linear regressions. J. Financ. Econ. 110 (1), 110–138.

Andersen, L., Duffie, D., Song, Y., 2019. Funding value adjustments. J. Finance 47 (1), 145–192.

Ang, A., Longstaff, F.A., 2013. Systemic sovereign credit risk: Lessons from the U.S. and Europe. J. Monetary Econ. 60, 493–510.

Bali, T., Heidari, M., Wu, L., 2009. Predictability of interest rates and interest-rate portfolios. J. Bus. Econom. Statist. 27, 517–527.

Cera, K., Molitor, P., Tsonchev, V., 2020. Some way to go in the transition to the €STR. https://www.ecb.europa.eu/pub/financial-stability/fsr/focus.

Cousin, A., Maatouk, H., Rullière, D., 2016. Kriging of financial term-structures. European J. Oper. Res. 255 (2), 631–648.

Doshi, H., Ericsson, J., Jacobs, K., Turnbull, S.M., 2013. Pricing credit default swaps with observable covariates. Rev. Financ. Stud. 26 (8), 2049–2094.

Duffee, G.R., 2013. Forecasting interest rates. In: Elliott, G., Timmermann, A. (Eds.), In: Handbook of Economic Forecasting, vol. 2, Elsevier, pp. 385–426, Chapter 7.

Duffie, D., 2018. Post-Crisis Bank Regulations and Financial Market Liquidity. In: Baffi Lecture at Banca D'Italia, https://www.darrellduffie.com/uploads/policy/DuffieBaffiLecture2018.pdf.

Filipović, D., Trolle, A.B., 2013. The term structure of interbank risk. J. Financ. Econ. 109, 707–733.
Heidari, M., Wu, L., 2010. Market anticipation of fed policy changes and the term structure of interest rates. Rev. Finance 14, 313–342.
Huang, W., Todorov, K., 2022. The post-Libor world: a global view from the BIS derivatives statistics. BIS Q. Rev. 19–32.
Jarrow, R., Li, H., Ye, X., Hu, M., 2019. Exploring mispricing in the term structure of CDS spreads. Rev. Finance 23 (1), 161–198.
Joslin, S., Singleton, K.J., Zhu, H., 2011. A new perspective on Gaussian dynamic term structure models. Rev. Financ. Stud. 24 (3), 926–970.
Klingler, S., Syrstad, O., 2021. Life after LIBOR. J. Financ. Econ. 141, 783–801.
Lloyd, S.P., 2020. Estimating nominal interest rate expectations: Overnight indexed swaps and the term structure. J. Bank. Financ. 119, 105915, 1-19.
Lloyd, S.P., 2021. Overnight indexed swap-implied interest rate expectations. Finance Res. Lett. 38, 101430.
Rebonato, R., Ronzani, R., 2021. Is convexity efficiently priced? Evidence from international swap markets. J. Empir. Financ. 63, 392–413.
Sundaresan, S., Wang, Z., Yang, W., 2016. Dynamics of the Expectation and Risk Premium in the OIS Term Structure. Indiana University Working paper.